The article analyses the approaches to the long-term projection of economic growth and its factors (labour, capital and total factor productivity) in the Republic of Belarus by 2050. The authors apply the idea of hybrid approach using production functions in order to forecast the economic growth by 2050 and compare their results with world well-known forecasting institutions. All forecasts of foreign centers are summarized in the consensus forecast. The potential growth of the Belarusian economy according to the hybrid model is quite significant, even considering the bad starting conditions of the last three years.

**Key words:** forecasting; economic growth; long-term projection; labour and human capital; GDP growth; total factor productivity; capital forecasting.

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From the time of A. Smith, D. Ricardo, T. Malthus, F. Ramsey, J. Schumpeter economist understood that to study the causes and sources of economic growth was extremely important. Investigation of issues related with economic growth is a significant branch of theoretical economy and forms the theory of economic growth.

Average annual historical Belarusian GDP growth in PPP terms and GDP per capita are presented in fig. 1 [3]. The aim of the article is to forecast the potential economic growth in the Republic of Belarus for the period 2016–2050 with the help of proposed hybrid long-term economic growth models which are based on six production functions and complemented by hybrid scenario forecasting models with changes in growth factors. The idea of hybrid growth models was borrowed from cybernetics and based on Shannon synthesis of reliable circuits from unreliable elements. The hybrid models integrate well-known production functions such as Cobb – Douglas, Solow, Mankiw – Romer – Weil, Schults and David – Klundert models. The hybrid models are applied to foretell the economic growth in 2015–2030–2050 for the Eurasian Economic Union countries [3]. The basis of the approach is the hybrid capital growth models based on the saving theories of F. Modigliani and J. Duesenberry and complemented by the models of transfer of savings into investments. We also take into account the TFP models (Solow residual) on the base of modern technological convergence theory that includes the study of the rate of diffusion technologies (E. Kähler, Q. Weng, J. Rattsa, H. Stokke) and the influence of the scientific and technical progress on the economic growth (D. Acemoglu, P. Aghion, E. Borenstein, P. Romer).

The model of economic growth of GDP is a function of $Y$ at time $t$ and depends on the following production factors:

$$
GDP(t) = Y(L(t), K(t), A(t)),
$$

where $L(t)$ is labour resources of a country, mainly taken with regard to their quality (human capital is measured as the mean years of schooling), frequently the quality of human capital $H(t)$ is taken as a separate

¹Professors S. R. Hausmann and C. Hidalgo suggested Economic Complexity Index that projects the economic growth. Later two scientists became to project the economic growth separately. The Republic of Belarus according to this index had risen 14 positions from 1998 to 2008 and took the 21st place in the world. From 2008 till 2014 Belarus lost its position and today it has the 30–31st places.
factor: \( K(t) \) is the accumulated capital (fixed assets); \( A(t) \) is total factor productivity (TFP), in fact it is the influence of scientific and technical progress on labour productivity. Sometimes the model is complemented by the energetic factor as at high energy prices a substantial part of the energy capacity of a country becomes a significant drag for the economic growth of a country \([4]\).

The growth function \( Y(...) \) can be expressed through the following well-known production functions:

\[
Y(t) = A(t)K^\alpha(t) L^{-\alpha}(t) \quad \text{(Cobb – Douglas),}
\]

\[
Y(t) = A(t)K^\alpha(t)(L(t)H(t))^{1-\alpha} = A(t)K^\alpha(t)L^{1-\alpha}(t)H^{1-\alpha}(t) \quad \text{(Denison),}
\]

\[
Y(t) = K^\alpha(t)(A(t)L(t))^{1-\alpha} \quad \text{(Solow),}
\]

\[
Y(t) = K^\alpha(t)H^\beta(t)(A(t)L(t))^{1-\alpha-\beta} \quad \text{(Mankiw – Romer – Weil),}
\]

\[
Y(t) = A(t)K^\alpha(t)L^\beta(t)H^\gamma(t), \quad \alpha + \beta + \gamma = 1 \quad \text{(Schultz),}
\]

\[
Y(t) = A(t)K^\alpha(t)L^{-\alpha}(t)\left[1 - \left(\frac{B(t)}{P_E}\right)^{\frac{\rho}{1-\rho}}\right]^{-\frac{1}{\beta}} \quad \text{(David – Klundert),}
\]

where \( \alpha \) – capital elasticity; \( \beta \) or \( (1-\alpha) \) – labour elasticity; \( \gamma \) – elasticity of human capital; \( \rho \) – comparative (with productivity) price of electricity; \( B(t) \) – measure of energy productivity; \( P_E \) – oil-price forecast.

For the convenience of calculation it is taken the logarithm of the production functions of economic growth of a country. For the Cobb – Douglas model the result is the following function:

\[
\ln \text{GDP}(t) = \ln A(t) + \alpha \ln K(t) + (1-\alpha) \ln L(t).
\]

Then subtracting this equation from the similar one for \( t + 1 \), we derive the difference equation:

\[
\ln \text{GDP}(t) = \Delta \ln A(t) + \alpha \Delta \ln K(t) + (1-\alpha) \Delta \ln L(t). \quad (1)
\]

The growth rate of any variable is the rate of change of the logarithm of this variable, i. e. \( \dot{X}(t) / X(t) \) equals to \( d \ln X(t) / dt \). Taking into account differential approximation or difference \( \Delta \ln X \approx \% \Delta X = \text{Growth}X \) the equation (1) can be converted into the following convenient formula for practical calculation (in \%):

\[
\text{Growth} \text{GDP}(t) = \text{Growth} A(t) + \alpha \text{Growth} K(t) + (1-\alpha) \text{Growth} L(t).
\]

Figure 2 shows decomposition of Belarusian GDP growth during 2000–2014 according to an IMF expert B. Bakker.
The hybrid production model is the linear combination of the mentioned models with the coefficient 1/6 (all 6 models are equal) [6]:

\[
\text{GDP}_{\text{hybrid production}}(t) = \frac{1}{6} \left( A(t) K^\alpha(t) L^{1-\alpha}(t) + \frac{1}{6} \left( A(t) K^\alpha(t) L^{\alpha}(t) H^{1-\alpha}(t) \right) + \frac{1}{6} \left( K^\alpha(t) H^\beta(t) \left( A(t) L(t) \right)^{1-\alpha} \right) + \frac{1}{6} \left( A(t) K^\alpha(t) L^{\alpha}(t) H^{\beta}(t) \right) + \frac{1}{6} \left( A(t) K^\alpha(t) L^{\alpha}(t) \right)^{1-\alpha} \left( H(t) \right)^\beta \right) + \frac{1}{6} \left( A(t) K^\alpha(t) L^{\alpha}(t) \right)^{1-\alpha} \left( H(t) \right)^\beta \left( 1 - \frac{B(t)}{P_E} \right) \right].
\]

The hybrid model is a multifactor model as contains the factors of economic growth from different models with weakened influence of separate factors because of the averaging.

The final version of the hybrid production model is as follows:

\[
\text{Growth GDP}_{\text{hybrid production}}(t) = \frac{5}{6} \text{Growth } A(t) + \frac{1}{3} \text{Growth } K(t) + \frac{5}{9} \text{Growth } L(t) + \frac{2}{9} \text{Growth } H(t) - \frac{1}{6,53} \text{Growth } L(t) \left( 1 - \frac{B(t)}{P_E} \right)^{\frac{6,53}{733}}.
\]  

(2)

Along with equation (2) we use its simplified version without taking into account the energy efficiency of the economy (fig. 3) or simplified version of the Cobb – Douglas function (fig. 4):

\[
\text{Growth GDP}_{\text{hybrid production}}(t) = \frac{5}{6} \text{Growth } A(t) + \frac{1}{3} \text{Growth } K(t) + \frac{5}{9} \text{Growth } L(t) + \frac{2}{9} \text{Growth } H(t).
\]  

(3)

It should be noted that the construction of the hybrid production model (2) or (3) is the first step. The main thing is to build hybrid models for forecasting of changes of production factors.
Even with the simplest predictions for long-term period the production factors can not be the same, they need to be recursively changed, perhaps every 5–10 years. The best results give time continuous functions of factor changes. The question is that there are numerous functions. Usually, each forecasting center has its own. Moreover, there are at least three scenarios (optimistic, pessimistic and mean). According to the hybrid approach such functions and forecasts should be averaged, focusing on the fact that an average scenario will be in the future.

**The hybrid model of long-term forecasting of labour force with regard to its quality**

In the first half of the XXI century will be a sharp slowdown in the growth of world population. If during 1980–2014 the average annual growth was 1.3 %, in the period 2015–2050 it will be only 0.5 % (fig. 5). The slowdown of average annual growth rate of working age population will be dramatic. It will fall from 1.7 to 0.3 %.

Life expectancy at birth of an average man on the earth in 2010–2015 will increase from 70.5 years up to 77.1 in 2045–2050. For Belarus this number will rise from 71.1 years to 75.1 in 2045–2050. The increase in life expectancy will lead to aging of world population. The mean age will rise from 39.6 to 42.2 years in 2050 and to 43.6 in 2100. Life expectancy of an average man on the earth will increase by 4.6 years, in Belarus by 3.4 years, so approximately for 1 year a Belarusian man lives less than the average man on the earth. The UN experts think that this difference will remain in 2100, though newly born Belarusians will live 87.4 years at that period (fig. 6).
For the economic growth the rate of growth/reduction of working age population is more important. Usually it’s characterized by the cluster of population at the age of 15–64. It is clear that only a certain amount of its share is employed. However, its annual growth rate is the same as the growth rate of the economically active population. In the models of economic growth on the forecasting horizon is expected steady growth (reduce) rate of labour force.

The UN [7] forecasts a high average annual rate of population decline (–0.57 %) for Belarus for the period 2000–2050 despite the increase in fertility rate. The EEC UN projects average annual decrease of population by –0.1 % by high-line scenario. According to the forecast Belarus is an outsider at an average annual rate of population growth and for the interval 2050–2100 this average annual growth rate is –0.73 %. Belarusian population will be reduced up to 6.9 million people by 2100. It seems that the UN does not take into account the efforts that are being made in our country in order to change the demographic situation.

The reduction of Belarusian population as a whole and economically active population are forecasted by the averaged data by the UN, the US Census Bureau and National Statistics (fig. 7). The population will decrease up to 9.0 million people by 2030 and up to 8.2 by 2050. The growth (reduction) rate of working age people will be 0.12 % for the period 2016–2050 according to the hybrid approach. The losses in this factor Belarus will compensate due to the labour force of pensioners and immigrants.

The reduction of labour force in Belarus for more than 20 % and increase of life expectancy even together with rise of pension age to 65 years for men and women will create a problem of reducing the share of the working population from 62 % in 2010 to 52 % in 2050. This factor makes difficulties with the financial support of aging people.

Many experts think that to create new work conditions for older people is the most effective and productive solution to the problem of aging population. It can occur in the compulsory form as in the EU (increasing the retirement age for men and women from 65 years to 69 years) or voluntarily as in Belarus (work with the preservation part of pension payments). Both approaches are justified by the medical progress that pushes the boundaries of working age as well as the influence of automation and computerization that improves working conditions. In addition, there must be taken measures that help to combine parenting and career.

The quality of human capital in Belarus and in world-leader country Norway is demonstrated in the fig. 8. It is vivid that the mean years of schooling is moving closer with the advanced countries in the world. The hybrid approach evaluates the quality of human capital based on the date on the mean years of schooling by Barro – Lee [9] for the population at age of 25 years old and over with the help of the Hall-Jones approach. The Hall – Jones approach was based on a survey of international assessments of yield of education in countries at different levels of economic development. During first four years of schooling is supposed that the yield is 13.4 %, during next four years the yield is 10.1 %. For education for more than eight years the average yield is 6.8 %. The hybrid approach for Belarus assumes the mean years of schooling for the population aged over 25 increases over time at a rate obtained by extrapolating data trend of the last 20 years. This allows Belarus to get closer to the US on an average level of human capital per a worker by 2050.
Hybrid-production model for long-term forecasting of economic growth

Application of the hybrid model of population data with averaging the international and national organizations and taking into account the duration of education improves the situation with labour force in Belarus. On the interval 2016–2050 instead of reduction approximately by 0.4 % per year we get a 0.12 % increase.

The hybrid model of long-term capital forecasting

The second factor of economic growth is capital. The projection is based on the speed of depreciation rate $\delta$ and scenario assumptions about investment rate $\text{Inv}(t)$ according to one of the following models:

$$K(t) = \text{Inv}(t-1)\text{GDP}(t-1) + (1-\delta)K(t-1),$$

(4)

$$K(t) = \text{Inv}(t)\text{GDP}(t) + (1-\delta)K(t-1).$$

(5)

Model (4) means that the investments and depreciation rate of the previous period create the capital of the following period. The second model (5) takes into account the depreciation rate of the previous period and assumes that the investments create the capital at the same period. The investment rate $\text{Inv}(t)$ is a share of GDP that is allocated on capital investments and in its turn depends on savings rate.

The aggregate hybrid model for capital factor looks in the following way:

$$K(t) = \frac{1}{2}(\text{Inv}(t-1)\text{GDP}(t-1) + \text{Inv}(t)\text{GDP}(t) + (1-\delta)K(t-1)).$$

(6)

Model (6) shows that the first half of the investments of the current period increases the capital of the same period and the second part of the investments creates the capital of the next period. The depreciation rate is taken from the previous period.

We need to know the equations of the capital growth rate for our calculations. For model (4) we have:

$$\text{Growth } K(t) = \frac{K(t+1) - K(t)}{K(t)} \cdot 100 = \frac{\text{Inv}(t+1)\text{GDP}(t+1)}{K(t)} - \delta.$$

For model (5) the equation is the following:

$$\text{Growth } K(t) = \frac{K(t+1) - K(t)}{K(t)} \cdot 100 = \text{Inv}(t)\text{GDP}(t) \cdot \frac{1}{K(t)} - \delta = \frac{\text{Inv}(t)}{k(t)} - \delta.$$

Aggregating both equations we have the hybrid model for the capital growth rate:

$$\text{Growth } K(t) = \text{Inv}(t)\frac{\text{GDP}(t)}{2K(t)} + \text{Inv}(t+1)\frac{\text{GDP}(t+1)}{2K(t)} - \delta_{\text{hybrid}}.$$

Here $k(t) = \frac{K(t)}{\text{GDP}(t)}$ shows the capital intensity of an economy. It is important to evaluate the initial capital.

The cost of fixed assets in different countries of the world is given by the World Bank. This coefficient for Belarus equals to 98 % of GDP at PPP, for Japan $k(t) = 4.2$ of GDP, in the USA it is 2.5 of GDP. The analysis shows that during 1995–2010 the fixed assets of Belarusian economy grew by 2.5 times, i.e. the average annual growth rate was 6.3 %. This level of growth rate was provided due to the high level of investment rate (fig. 9).

The hybrid depreciation rate $\delta_{\text{hybrid}}$ is the average of the rates that were used by different forecasters: 4 % (GS, World Bank), 4.5 % (Carnegie), 5 % (PwC), 6 % (CEPII).
The hybrid approach requires not only savings averaging models by F. Modigliani and J. Duesenberry (as it is made at source [6]), but also multiple scenario hypothesis about the future of investment process that is represented as a piece-constant and piece-linear functions of investment rates. The hybrid model of the investment rate has the following form:

\[ \text{Inv}_{\text{hybrid}}(t) = \frac{1}{6} \text{Inv}_{\text{Duesenberry}}(t) + \frac{1}{6} \text{Inv}_{\text{Modigliani}}(t) + \frac{1}{6} \text{Inv}_{\text{Carnegie}}(t) + \frac{1}{6} \text{Inv}_{\text{CEPII}}(t) + \frac{1}{6} \text{Inv}_{\text{COE}}(t) + \frac{1}{6} \text{Inv}_{\text{PC}}(t). \]

Here \( \text{Inv}_{\text{Duesenberry}} \) and \( \text{Inv}_{\text{Modigliani}} \) mean functions that are obtained with the help of Feldstein – Harioka model of transformation saving into investments, where savings are defined by Duesenberry and Modigliani’s models respectively [7].

The forecast of investment rate changes by the hybrid approach showes that it will be very difficult for Belarus to keep the existing investment rate (30.9 %). Most foreign experts take the investment rate for Belarus at the level of 25 % in base-line scenario and 28 % in high-line scenario of growth. It must be said that there is a significant reserve for investment growth due to foreign capital in the privatization process in Belarus. Note that foreign capital provides a considerable investment growth in a short period of privatization. In long-run period it is known a worldwide proportion of national and foreign capital which equals to 9 : 1 (look World Investment Report).

As the result the hybrid model gives the average annual capital growth in Belarus 5.1 % during 2016–2050 (according to CEPII [4] it is about 5.3 %).

The hybrid model of long-term forecasting of total factor productivity

The growth projections of \( A(t) \) depend on the speed of technological convergence of national economies. To be exact, it depends on success of modernization process in a country. Many experts are based on catch-up modernization. According to this process the growth rate \( A(t) \) is slowing down as it gets the level of GDP per capita close to the US GDP.

The hybrid approach synthesizes two classes of convergence models. The first class is based on evaluation of the speed of catch-up modernization that is related to the degree of the country’s lag behind the technological leader. In empirical researches the technological leader is usually the US. The convergence speed depends on two factors in catch-up modernization process: on the rate of technological progress in the innovation-frontier countries and on the rate of technological leap of the country. The main scenarios of TFP growth \( A_i(t) \) of a country \( i \) are generated by different hypothesis about speed of technological convergence \( \beta_i \) of a country \( i \), by which is meant the borrowing technological rate of the country and its technology approximation (catch-up modernization) to technology leaders. According to this approach the speed of technological convergence \( \beta_i \) is the borrowing technology rate, the latter in its turn depends on the convergence conditions index (CCI), i. e. it depends on a number institutional factors which characterize the innovation system of a country.

TFP depends on how far it lags behind the technological leader (USA) and therefore has the potential for “catch-up modernization” by borrowing technologies. TFP is connected with the level of human capital and other, more institutional factors, such as political stability, openness-to-trade and foreign investments, stability.
of law, the strength of the financial system and the business culture of the business environment. The latest institutional factors are difficult to measure in quantitative terms through a single index, but are reflected in the assumptions about the relative speed of technological progress in each country, which we will calculate from the values of innovation and other rating indices.

The Nobel Prize winner in economics E. Phelps together with R. Nelson proposed a model (the Nelson – Phelps model) in which it is assumed that the growth of TFP \( A(t) \) depends both on the level of education in the country and on the difference between theoretically possible technological level \( T(t) \) (if all scientific discoveries were introduced immediately) and its true value:

\[
Growth \ A(t) = A(t) - \frac{T(t) - A(t)}{A(t)},
\]

where \( g(H) \) is a growth component of TFP that is explained by the level of education \( H(t) \) and is an increasing function; \( c(H) \) is an increasing function which depends on the level of education and is determined the rate of technology diffusion \( n \).

Application of the Nelson – Phelps model simplifies the Benhabib-Spiegel model by replacing a hard computable indicator \( T(t) \) into the indicator \( A(t) \) at the leader-country of innovation frontier (usually USA):

\[
Growth \ A'(t) = \frac{A'(t)}{A(t)} = g'(H'(t)) + c'(H'(t)) \left( \frac{A^{US}(t)}{A(t)} - 1 \right),
\]

where \( A^{US}(t) \) is TFP at the world leader country, i. e. the USA; summand \( c'(H'(t)) \left( \frac{A^{US}(t)}{A(t)} - 1 \right) \) represents the share of technological diffusion from the leader country to the country \( i \). It is assumed that the function \( C(\bullet) \) is increasing.

Simplifying, the growth of the country’s TFP can be determined from the dynamic equation (in %):

\[
Growth \ A'(t) = 1.33 - \beta \left( \ln GDP_{p,e}(t-1) - \ln GDP_{p,e}^{US}(t-1) \right).
\]

Here 1.33 % is a growth rate of TFP \( A^{US}(t) \) at the USA, \( \beta \) is a coefficient of \( \beta \)-convergence, that is usually equals 1.5 in base-line scenario \([8]\).

We use more complex model in order to calculate the speed of convergence \( \beta \) in a country \( i \):

\[
\beta_i = \begin{cases} 
\frac{(-CCI^i)^{1.5}}{-800} + 0.015, & \text{если } CCI < 0, \\
0.015, & \text{если } CCI \geq 0,
\end{cases}
\]

where the conditional convergence index \( CCI^i \) is the sum of scaled values of the most important rating innovation indices: Knowledge Economy Index (World Bank), Global Innovation Index (INSEAD), ICT Development Index (ITU), Doing Business (World Bank).

It is important to emphasize that the hybrid approach is intended only to obtain long-term forecasts. The approach ignores the cyclical fluctuations around a given long-term trend, ignores the possibility of serious adverse shocks (force majeure, such as political revolutions, natural disasters and military conflicts). At the same time the hybrid approach does not take into account the possibility of a sudden jump in technology due to some new major innovation, either through new breakthrough discoveries or innovative applications of existing technologies.

The second class of models is based on econometric regressions of TFP growth that characterize the quality of innovation system of a country. Many forecasts assume that Belarus will be able to implement “overtaking modernization” after 2030 due to accumulated scientific potential, high literacy of the population, i. e. to combine the investment and innovation phases of economies (according to Porter). One of indirect confirmation of this is the relatively high place of the country in Knowledge Economy Index, although the return on the available scientific potential is not high (79th place in Global Innovation Index, https://www.globalinnovation-index.org).

The CEPII \([4]\) forecasts that the speed in the countries of innovation frontier according to base-line scenario is 1.5 %, according to pessimistic scenario is 1 % and 2 % – in optimistic scenario. The Carnegie Foundation projects the CCI at the level of 2.3 % in China, 3.52 % in India, 2.54 % in Russia.

All projections of TFP growth are shown at fig. 10.
Long-term forecast of economic growth in Belarus

Forecasts of foreign centers [4; 11–13] are summarized in the consensus forecast as the arithmetic mean and are represented on fig. 11 and 12. The forecast according to the hybrid-production model is made on three versions of models, to be exact according to models (2)–(3) and then averaged. It also shows the result of the calculation of the hybrid model. The potential growth of the Belarusian economy according to the hybrid model is quite significant, even considering the bad starting conditions of the last three years. In the case of GDP growth by 3.9 % annually during 25 years, the Belarusian economy will grow by 2.6 times and the welfare will increase by 2.7 times.

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**Fig. 10.** Comparison of annual TFP growth forecasts for Belarus on the interval 2016–2050 with the result of the hybrid model


**Fig. 12.** The forecast of average annual welfare growth (GDP per capita at PPP) in Belarus in 2016–2050 (notes: EEC 2000–2040 (high-line scenario); CEPII 2013–2050 [3]; HSBC (governments make complete progress) 2010–2050 [11]; Harvard 2014–2024 [12])
Potential economic growth (it does not mean real) of Belarusian GDP and welfare in the nearest 35 years might be significant. Despite the demographic problems it might be supported due to traditionally high level of investments, especially if to take geographical advantages of the Republic of Belarus, and dynamic technology adoption with its own innovation implementation. The dynamics of the latter can be high because of the high quality of education level.

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