

## THE DYNAMICS OF THE ENTERPRISE'S INCOME IN THE EXTENSIVE METHOD OF DEVELOPMENT

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The paper explores the possibilities of using extended production and trade in the commodity market. The results are obtained on the basis of a certain economic-mathematical model that allows to express the producer's net proceeds through essential market parameters. These include: the price and volume of sales of goods or services, the absolute value of the coefficient of price elasticity of demand, the inflation rate, the coefficient of production costs and the level of the tax rate. The strengths and weaknesses of trade in the conditions of extended production are revealed. An analysis of the conditions which, on the basis of the law of demand, ensure the growth of net proceeds from the sale of products is given. The best ways of implementing the extensive way of enterprise development are indicated.

**Key words:** market; goods and services; proceeds; net proceeds; tax rate.

## О ДИНАМИКЕ ДОХОДА ПРЕДПРИЯТИЯ ПРИ ЭКСТЕНСИВНОМ ПУТИ РАЗВИТИЯ

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Исследуются возможности использования расширенного производства и торговли на товарном рынке. Результаты получены на основе построения определенной экономико-математической модели, позволяющей выразить выручку-нетто производителя через существенные параметры рынка, а именно: цену и объем продаж товара или оказываемой услуги, абсолютную величину коэффициента ценовой эластичности спроса, коэффициент инфляции за рассматриваемый период времени, коэффициент издержек производства и уровень налоговой ставки. Выявлены сильные и слабые стороны торговли в условиях расширенного производства. Проанализированы условия, которые на основе закона спроса обеспечивают рост выручки-нетто от реализации продукции. Указаны наилучшие способы реализации экстенсивного пути развития предприятия.

**Ключевые слова:** рынок; товары и услуги; выручка; выручка-нетто; налоговая ставка.

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## Introduction

The paper is devoted to the study of one of the possible methods of production and trade in the market of goods or services, set out in [1, p. 115; 2, p. 125] and [3], which is called the extensive method. We study a simple situation of increasing the output and realization of goods without special price changes using a certain economic-mathematical model. Unlike the available results, the dependence of the manufacturer's net proceeds on the model parameters, including the advantages and disadvantages of this approach, as well as the benefits and prospects of trade are analysed. By *net proceeds* we mean all gross proceeds that the entrepreneur receives on the basis of concluded contracts, minus the taxes calculated from it [4, p. 122].

Let the manufacturer sells on the market for a certain period of time  $q$  units of goods at a price  $p$  per unit. Then the proceeds from the sale will be  $qp$  monetary units. Let the tax rate on this amount according to the current legislation is  $i$ , where  $0 < i < 1$ , i. e. net proceeds of the seller are the value  $qp(1 - i)$ .

## Extensive method

In the method, which is based on the increase in output [2], proceeds are determined by the expression:

$$R_1 = q(p + \sigma) + \Delta q_1(p + \sigma)(1 - a) = (q + \Delta q_1(1 - a))(p + \sigma),$$

where the following notations are used:  $q$  and  $p$  – sales volume and price;  $\Delta q_1$  – changes in sales;  $\sigma$  – background inflation in monetary terms per unit of output;  $a$  ( $0 < a < 1$ ) – coefficient of production costs in the production of an additional unit of product.

In the paper [2] the formula (1.17) of dependence of sales volume  $q = q(p)$  from price  $p$  is formulated. By using this formula and expressing on this basis parameter  $p$ , the formula of proceeds is

$$R_1 = (q + \Delta q_1(1 - a)) \left( p + \sigma - p \frac{\Delta q_1}{eq} \right),$$

where  $e > 0$  – absolute value of the price elasticity of demand.

To further analyse the dependence of the proceeds on the parameters of the model, we introduce the following quantities:

$$k_q = \frac{\Delta q_1}{q}, \quad k_p = \frac{\sigma}{p}.$$

Let's call them, respectively, the coefficient of changes in sales and inflation rate. Then the proceeds will take the following form:

$$R_1 = qp(1 + k_q(1 - a))(e(1 + k_p) - k_q)/e. \quad (1)$$

Suppose that  $i$  is the initial share of total payments from proceeds to obtain the entrepreneur's net proceeds, and  $i_1$  means the corresponding share of total payments from the changed proceeds. Obviously, taking into account the tax scale, the inequality  $i_1 \geq i$  will be fulfilled. If an extensive method of trading is used, net proceeds of the entrepreneur will be expressed by the formula  $R_1(1 - i_1)$ , where  $R_1$  is a seller's proceeds in the extensional method [3]. If we substitute here the formula for proceeds (1), we obtain the formula for net proceeds  $C_1$ :

$$C_1 = qp(1 + k_q(1 - a))(e(1 + k_p) - k_q)(1 - i_1)/e. \quad (2)$$

The initial net proceeds are formed from the initial gross proceeds  $qp(1 + k_p)$  (see [3]) and the tax rate. The increase in net proceeds by using an extensive method in comparison with the original one, which is equal to  $qp(1 + k_p)(1 - i)$ , will occur when the following inequality is executed:

$$C_1 > qp(1 + k_p)(1 - i).$$

Let's explore this condition. We substitute expression (2) and obtain

$$qp(1 + k_q(1 - a))(e(1 + k_p) - k_q)(1 - i_1)/e > qp(1 + k_p)(1 - i).$$

We consistently simplify this inequality. Then we obtain the relation

$$(1 + k_q(1 - a))(e(1 + k_p) - k_q)(1 - i_1) > e(1 + k_p)(1 - i).$$

We introduce the following notation for the convenience of studying the above inequality:

$$x = k_q, \quad \bar{e} = e(1 + k_p), \quad I = (1 - i)/(1 - i_1).$$

Then the investigated inequality of increase of net proceeds leads to an inequality with the unknown variable  $x$  in the form:

$$(1 + x(1 - a))(\bar{e} - x) > \bar{e}I, \quad x \geq 0.$$

Then we get the following:

$$(1 + x(1 - a))(\bar{e} - x) > \bar{e}I \Leftrightarrow -x^2(1 - a) - x(\bar{e}(1 - a)) + \bar{e}(1 - I) > 0.$$

As a result, we have the inequality:

$$x^2(1 - a) - x(\bar{e}(1 - a) - 1) + \bar{e}(I - 1) < 0. \quad (3)$$

Remember, that  $i_1 \geq i$ , and since  $I = (1 - i)/(1 - i_1)$ , we have the inequality  $I > 1$ . Let's analyse the quadratic trinomial in expression (3). For this purpose we write out its discriminant:

$$D = (\bar{e}(1 - a) - 1)^2 - 4\bar{e}(1 - a)(I - 1).$$

We consider two cases.

1. Suppose first that  $D > 0$ . Then we have two solutions:

$$x_1 = \frac{(\bar{e}(1 - a) - 1) - \sqrt{(\bar{e}(1 - a) - 1)^2 - 4\bar{e}(1 - a)(I - 1)}}{2(1 - a)},$$

$$x_2 = \frac{(\bar{e}(1 - a) - 1) + \sqrt{(\bar{e}(1 - a) - 1)^2 - 4\bar{e}(1 - a)(I - 1)}}{2(1 - a)}.$$

Both roots are positive. In this case, the solution of inequality (3) is the interval  $x_1 < x < x_2$ .

2. If we have the inequality  $D \leq 0$ , then taking into account the assumption  $1 - a > 0$  the inequality (3) doesn't have solutions.

The result of the research leads to the conclusion that the growth of net proceeds using an extensive method will be if and only if the coefficient of changes in sales  $k_q$  is subject to the following condition:

$$(\bar{e}(1 - a) - 1)^2 - 4\bar{e}(1 - a)(I - 1) > 0, \quad x_1 < k_q < x_2.$$

**The case  $i_1 = i$ .** Here  $I = 1$  and the roots of the square trinomial (3) take the following form:

$$x_1 = 0, \quad x_2 = (\bar{e}(1 - a) - 1)/(1 - a).$$

From the previous arguments it follows, that when  $i_1 = i$  conditions for grows of net proceeds are equivalent to the requirement of one inequality:

$$0 < k_q < (\bar{e}(1 - a) - 1)/(1 - a). \quad (4)$$

### Optimization of net proceeds

We need to calculate the maximum of the net proceeds function (2) with respect to the variable  $x = k_q$ , assuming that condition (4). To do this, we formulate the following nonlinear programming problem:

$$\begin{cases} C_1(x) = qp(1 + x(1 - a))(\bar{e} - x)(1 - i_1)/e \rightarrow \max, \\ 0 \leq x \leq x_2, \\ \bar{e}(1 - a) - 1 > 0. \end{cases} \quad (5)$$

We solve the problem (5) graphically. For this case we construct a graph of the function  $C_1(x)$ . First we transform the function  $C_1(x)$  as follows:

$$\begin{aligned} C_1(x) &= qp(1+x(1-a))(\bar{e}-x)(1-i_1)/e = \\ &= qp(\bar{e} + \bar{e}x(1-a) - x - x^2(1-a))(1-i_1)/e = \\ &= qp(-x^2(1-a) + x(\bar{e}(1-a)-1) + \bar{e})(1-i_1)/e. \end{aligned}$$

This is a quadratic function, in which the branches of the parabola are directed downward. Let's find the coordinate  $x = x^*$  of the top of the parabola with respect to the variable  $x$  of the function  $C_1(x)$ , calculating the derivative and equating it to zero:

$$\begin{aligned} \frac{dC_1(x)}{dx} &= qp(-2x(1-a) + \bar{e}(1-a) - 1)(1-i_1)/e = 0, \\ -2x(1-a) &= 1 - \bar{e}(1-a) \Rightarrow x^* = \frac{1}{2}\left(\bar{e} - \frac{1}{1-a}\right). \end{aligned}$$

Using the second restriction presented in (5), we obtain the inequality  $\bar{e} > 1/(1-a)$ , where  $\bar{e} > 0$  and  $0 < a < 1$ , which means  $x^* > 0$ . Now we calculate the value of the function  $C_1(x)$  at the point  $x^*$ :

$$\begin{aligned} C_1(x^*) &= qp\left(-\left(\frac{1}{2}\left(\bar{e} - \frac{1}{1-a}\right)\right)^2(1-a) + \frac{1}{2}\left(\bar{e} - \frac{1}{1-a}\right)(\bar{e}(1-a)-1) + \bar{e}\right)(1-i_1)/e = \\ &= qp\left(-\frac{1}{4}\left(\bar{e}^2(1-a) + \frac{1}{1-a} - 2\bar{e}\right) + \frac{1}{2}\left(\bar{e}^2(1-a) - 2\bar{e} + \frac{1}{1-a}\right) + \bar{e}\right)(1-i_1)/e = \\ &= \frac{1}{2}qp\left(-\frac{1}{2}\bar{e}^2(1-a) - \frac{1}{2(1-a)} + \bar{e} + \bar{e}^2(1-a) - 2\bar{e} + \frac{1}{1-a} + 2\bar{e}\right)(1-i_1)/e = \\ &= \frac{1}{2}qp\left(\frac{1}{2}\bar{e}^2(1-a) + \frac{1}{2(1-a)} + \bar{e}\right)(1-i_1)/e. \end{aligned} \tag{6}$$

It is clear, that  $C_1(x^*) > 0$ . Let's calculate the roots of the quadratic function  $C_1(x)$ . Its discriminant has the form

$$\begin{aligned} D &= (\bar{e}(1-a)-1)^2 + 4\bar{e}(1-a) = \\ &= \bar{e}^2(1-a)^2 + 1 + 2\bar{e}(1-a) = (\bar{e}(1-a)+1)^2. \end{aligned}$$

As a result, we get the following roots:

$$x_1 = -1/(1-a), x_2 = \bar{e}.$$

When the graph of the function  $C_1(x)$  intersects the axis  $Oy$  we have  $C_1(0) = qp\bar{e}(1-i_1)/e$ . The graph is based on all the data obtained above and shown in fig. 1.

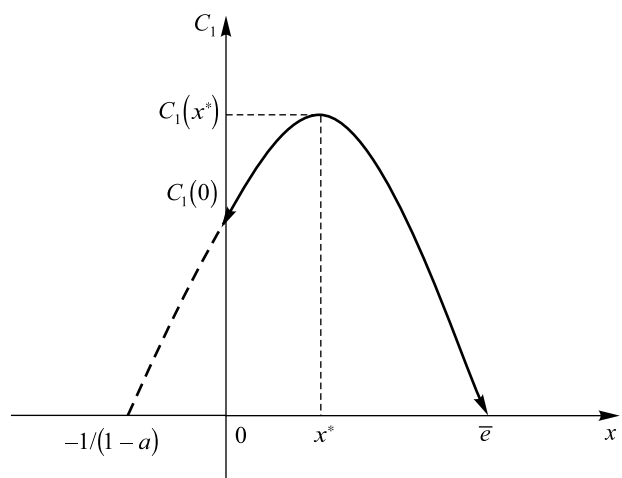


Fig. 1. Net proceeds function graph  $C_1 = C_1(x)$

**Research of the dependence of the function  $C_1(x^*)$  on various parameters**

**Dependence on the parameter  $i_1$ .** Using equality (6), we transform the function  $C_1(x^*)$  of the parameter  $i_1$  taking into account that  $0 < i_1 < 1$ . In this case, we replace the value  $\bar{e}$  with  $e(1 + k_p)$ . So we have

$$\begin{aligned} \omega(i_1) &= \frac{1}{2}qp \left( \frac{1}{2}e^2(1 + k_p)^2(1 - a) + \frac{1}{2(1 - a)} + e(1 + k_p) \right) (1 - i_1)/e = \\ &= \frac{1}{2}qp \left( \frac{1}{2}e(1 + k_p)^2(1 - a) + \frac{1}{2e(1 - a)} + (1 + k_p) \right) (1 - i_1), \quad \bar{e}(1 - a) - 1 > 0, \end{aligned} \tag{7}$$

where  $\omega(i_1)$  is a linear function of the argument  $i_1$ . We note, that for  $i_1 \rightarrow +\infty$  it follows:  $\omega(i_1) \rightarrow -\infty$ , and for  $i_1 \rightarrow -\infty$  occurs:  $\omega(i_1) \rightarrow +\infty$ . Then we calculate the two limits in the case when  $i_1 \rightarrow +0$  and  $i_1 \rightarrow 1$ :

$$\bar{\omega} = \lim_{i_1 \rightarrow +0} \omega(i_1) = \frac{1}{2}qp \left( \frac{1}{2}e(1 + k_p)^2(1 - a) + \frac{1}{2e(1 - a)} + (1 + k_p) \right), \quad \lim_{i_1 \rightarrow 1} \omega(i_1) = 0.$$

The maximum value of the function  $\omega(i_1)$  is achieved at  $i_1 \rightarrow 0$ , in the case of an increase in the share of total payments from proceeds the amount of net proceeds naturally decreases.

On the basis of the above data, we represent the graph of the function  $\omega(i_1)$  in fig. 2.

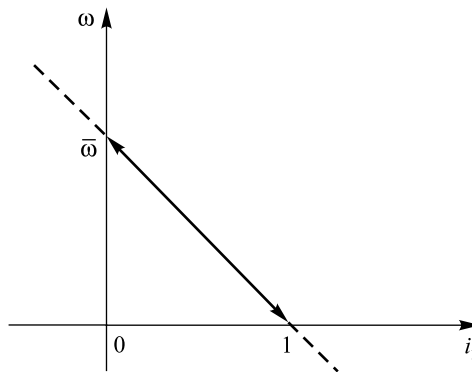


Fig. 2. Net proceeds function graph  $\omega = \omega(i_1)$

**Dependence on the parameter  $k_p$ .** We express the function represented in formula (7) as  $\beta(k_p)$ , which is defined for  $\bar{e}(1 - a) - 1 > 0$ , i. e. for  $k_p > (1 - e(1 - a))/e(1 - a)$  we have

$$\begin{aligned} \beta(k_p) &= \frac{1}{2}qp \left( \frac{1}{2}e(1 + k_p)^2(1 - a) + \frac{1}{2e(1 - a)} + (1 + k_p) \right) (1 - i_1) = \\ &= \frac{1}{2}qp \left( \frac{1}{2}e(1 - a) + \frac{1}{2}k_p^2e(1 - a) + k_p e(1 - a) + \frac{1}{2e(1 - a)} + 1 + k_p \right) (1 - i_1) = \\ &= \frac{1}{2}qp \left( \frac{1}{2}k_p^2e(1 - a) + k_p (e(1 - a) + 1) + \frac{1}{2}e(1 - a) + \frac{1}{2e(1 - a)} + 1 \right) (1 - i_1). \end{aligned}$$

The function  $\beta(k_p)$  is quadratic, its graph is a parabola, the branches of which are directed upwards. To determine the coordinates of the top of the parabola, we calculate the derivative of the given function and equate it to zero. So we have

$$\frac{d\beta(k_p)}{dk_p} = \frac{1}{2}qp (k_p e(1 - a) + e(1 - a) + 1) (1 - i_1) = 0,$$

$$k_p e(1-a) = -e(1-a) - 1 \Leftrightarrow k_p^* = -\left(1 + \frac{1}{e(1-a)}\right).$$

Now we calculate  $\beta(k_p)$  at the point  $k_p = k_p^*$ :

$$\begin{aligned} \beta(k_p^*) &= \frac{1}{2}qp \left( \frac{1}{2} \left( -1 - \frac{1}{e(1-a)} \right)^2 e(1-a) - \left( 1 + \frac{1}{e(1-a)} \right) (e(1-a) + 1) \right) (1-i_1) + \\ &\quad + \frac{1}{2}qp \left( \frac{1}{2}e(1-a) + \frac{1}{2e(1-a)} + 1 \right) (1-i_1) = \\ &= \frac{1}{2}qp \left( \frac{1}{2} \left( 1 + \frac{1}{e^2(1-a)^2} + \frac{2}{e(1-a)} \right) e(1-a) - e(1-a) \right) (1-i_1) + \\ &\quad + \frac{1}{2}qp \left( -2 - \frac{1}{e(1-a)} + \frac{1}{2}e(1-a) + \frac{1}{2e(1-a)} + 1 \right) (1-i_1) = \\ &= \frac{1}{2}qp \left( \frac{1}{2}e(1-a) + \frac{1}{2e(1-a)} + 1 - \frac{1}{2}e(1-a) - \frac{1}{2e(1-a)} - 1 \right) (1-i_1) = 0. \end{aligned}$$

Taking into account the condition  $k_p > (1 - e(1-a))/e(1-a) = \bar{k}_p > 0$  the graph of the function  $\beta(k_p)$  is shown in fig. 3, where is set the following:

$$\begin{aligned} \beta(0) &= \frac{1}{2}qp \left( \frac{1}{2}e(1-a) + \frac{1}{2e(1-a)} + 1 \right) (1-i_1), \quad \hat{e} = e(1-a); \\ \beta(\bar{k}_p) &= \frac{qp}{2} \left( \frac{1}{2} \left( \frac{1-\hat{e}}{\hat{e}} \right)^2 \hat{e} + \frac{1-\hat{e}}{\hat{e}} (\hat{e}+1) + \frac{1}{2}\hat{e} + \frac{1}{2\hat{e}} + 1 \right) (1-i_1) = \\ &= \frac{qp(1-i_1)}{2} \left( \frac{\hat{e}}{2} \left( \frac{1}{\hat{e}^2} + 1 - \frac{2}{\hat{e}} \right) + 1 - \hat{e} + \frac{1}{\hat{e}} - 1 + \frac{\hat{e}}{2} + \frac{1}{2\hat{e}} + 1 \right) = \\ &= \frac{qp(1-i_1)}{2} \left( \frac{1}{2\hat{e}} + \frac{\hat{e}}{2} - 1 - \frac{\hat{e}}{2} + \frac{3}{2\hat{e}} + 1 \right) = \frac{qp(1-i_1)}{2} \frac{2}{\hat{e}} = \frac{qp(1-i_1)}{\hat{e}}. \end{aligned}$$

Based on the above graph, it can be concluded that the function of the entrepreneur's net proceeds grows with an increase in the inflation rate  $k_p$ .

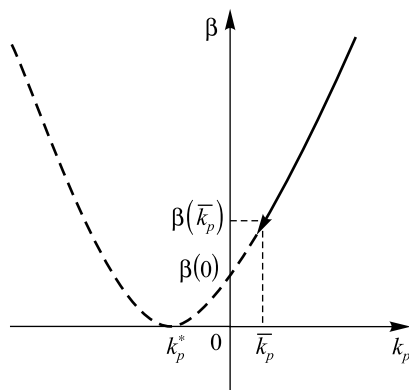


Fig. 3. Net proceeds function graph  $\beta = \beta(k_p)$

**Dependence on the parameter  $e$ .** Now we express the function represented in formula (7) as  $\gamma(e)$ , which is defined for  $\bar{e}(1-a) - 1 > 0$ , i. e. for  $e > 1/(1-a)(1+k_p) = \tilde{e} > 0$ :

$$\gamma(e) = \frac{1}{2}qp \left( \frac{1}{2}e(1+k_p)^2(1-a) + \frac{1}{2e(1-a)} + (1+k_p) \right) (1-i_1).$$

It should be noted that when  $e \rightarrow 1/(1-a)(1+k_p) + 0$  the function  $\gamma(e) \rightarrow qp(1+k_p)(1-i_1)$ , and when  $e \rightarrow \infty$  the function  $\gamma(e) \rightarrow \infty$ . So we calculate the derivative and equate it to zero:

$$\frac{d\gamma(e)}{de} = \frac{1}{2}qp \left( \frac{1}{2}(1+k_p)^2(1-a) - \frac{1}{2e^2(1-a)} \right) (1-i_1) = 0,$$

$$\frac{1}{2e^2(1-a)} = \frac{1}{2}(1+k_p)^2(1-a) \Leftrightarrow e^2 = \frac{1}{(1+k_p)^2(1-a)^2} \Leftrightarrow e_{1,2}^* = \pm \frac{1}{(1+k_p)(1-a)}.$$

Next, we define the value of the function  $\gamma(e)$  at the points  $e_1^* = \tilde{e}$  and  $e_2^*$ :

$$\begin{aligned} \gamma(e_1^*) &= \frac{1}{2}qp \left( \frac{(1+k_p)^2(1-a)}{2(1+k_p)(1-a)} + \frac{(1+k_p)(1-a)}{2(1-a)} + 1+k_p \right) (1-i_1) = \\ &= \frac{1}{2}qp \left( \frac{(1+k_p)}{2} + \frac{(1+k_p)}{2} + 1+k_p \right) (1-i_1) = qp(1+k_p)(1-i_1), \\ \gamma(e_2^*) &= \frac{1}{2}qp \left( -\frac{(1+k_p)^2(1-a)}{2(1+k_p)(1-a)} - \frac{(1+k_p)(1-a)}{2(1-a)} + 1+k_p \right) (1-i_1) = \\ &= \frac{1}{2}qp \left( -\frac{(1+k_p)}{2} - \frac{(1+k_p)}{2} + 1+k_p \right) (1-i_1) = 0. \end{aligned}$$

Taking into account the requirements  $k_p > 0$  and  $0 < i_1 < 1$ , we obtain the inequality  $\gamma(e_1^*) > 0$ . The graph of the function  $\gamma(e)$  is shown in fig. 4.

Analysing fig. 4, we can conclude that the net proceeds function  $\gamma(e)$  increases with  $e > e_1^*$ .

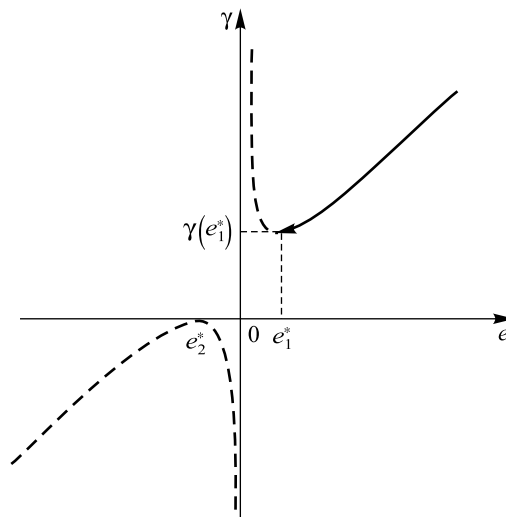


Fig. 4. Net proceeds function graph  $\gamma = \gamma(e)$

**Dependence on the parameter  $a$ .** We transform expression (7) as a function  $\varphi(a)$  for the value  $0 < a < 1$ , which is defined for  $\bar{e}(1-a) - 1 > 0$ , i. e. for  $a < (\bar{e} - 1)/\bar{e}$ ,  $\bar{e} > 1$ . So we have the following:

$$\varphi(a) = \frac{qp(1-i_1)}{2} \left( \frac{1}{2} e(1+k_p)^2 - \frac{1}{2} e(1+k_p)^2 a + \frac{1}{2e(1-a)} + (1+k_p) \right).$$

The function  $\varphi(a)$  presented above has the following properties: when  $a \rightarrow \infty$  then  $\varphi(a) \rightarrow -\infty$ ; when  $a \rightarrow -\infty$  then  $\varphi(a) \rightarrow \infty$ . At the point  $a = 0$  we get:

$$\varphi(0) = \frac{qp(1-i_1)}{2} \left( \frac{1}{2} e(1+k_p)^2 + \frac{1}{2e} + (1+k_p) \right) > 0.$$

The coordinate of the extremal point of the function graph  $\varphi(a)$  with respect to the variable  $a$  is calculated as follows:

$$\begin{aligned} \frac{d\varphi(a)}{da} &= \frac{qp(1-i_1)}{2} \left( -\frac{1}{2} e(1+k_p)^2 + \frac{1}{2e(1-a)^2} \right) = 0, \\ \frac{1}{2e(1-a)^2} &= \frac{1}{2} e(1+k_p)^2 \Leftrightarrow (1-a)^2 = \frac{1}{e^2(1+k_p)^2} \Leftrightarrow |1-a| = \frac{1}{e(1+k_p)}, \\ a_1^* &= 1 - \frac{1}{e(1+k_p)}, \quad a_2^* = 1 + \frac{1}{e(1+k_p)}. \end{aligned}$$

The values of the function  $\varphi(a)$  at the points  $a_1^*$  and  $a_2^*$  are respectively equal to

$$\begin{aligned} \varphi(a_1^*) &= \frac{qp(1-i_1)}{2} \left( \frac{1}{2} e(1+k_p)^2 - \frac{1}{2} e(1+k_p)^2 \left( 1 - \frac{1}{e(1+k_p)} \right) \right) + \\ &+ \frac{qp(1-i_1)}{2} \left( 1 / \left( 2e \left( 1 - 1 + \frac{1}{e(1+k_p)} \right) \right) + (1+k_p) \right) = \\ &= \frac{qp(1-i_1)}{2} \left( \frac{e(1+k_p)^2}{2e(1+k_p)} + \frac{(1+k_p)}{2} + (1+k_p) \right) (1-i_1) = qp(1+k_p)(1-i_1); \\ \varphi(a_2^*) &= \frac{qp(1-i_1)}{2} \left( \frac{1}{2} e(1+k_p)^2 - \frac{1}{2} e(1+k_p)^2 \left( 1 + \frac{1}{e(1+k_p)} \right) \right) + \\ &+ \frac{qp(1-i_1)}{2} \left( 1 / \left( 2e \left( 1 - 1 - \frac{1}{e(1+k_p)} \right) \right) + (1+k_p) \right) = \frac{qp(1-i_1)}{2} \left( -\frac{e(1+k_p)^2}{2e(1+k_p)} - \frac{(1+k_p)}{2} + (1+k_p) \right) = 0. \end{aligned}$$

Since  $k_p > 0$  and  $0 < i_1 < 1$ , so we have  $\varphi(a_1^*) > 0$ .

Taking into account the conditions  $a < (\bar{e} - 1)/\bar{e}$  and  $0 < a < 1$ , we obtain  $e(1+k_p) > 1$ . Then we have  $a_1^* > 0$  and  $a_2^* > 0$ . The graph of the function  $\varphi(a)$  is shown in fig. 5.

The function has economic meaning only in the area of  $0 < a < a_1^*$ , since by condition  $a < 1 - 1/e(1+k_p)$ . In this case, the value of net proceeds  $\varphi(a)$  falls on a given interval with the increase of the parameter  $a$ .



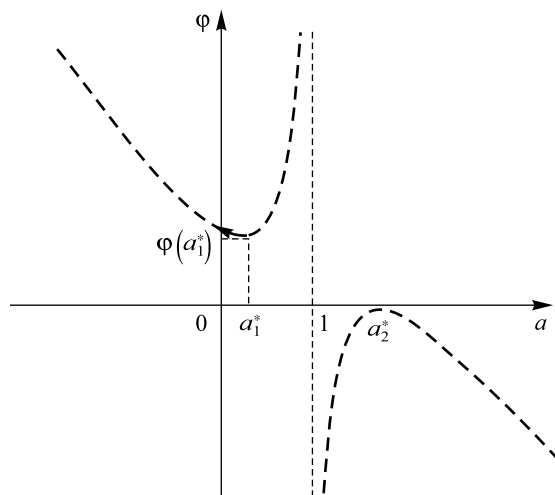


Fig. 5. Net proceeds function graph  $\varphi = \varphi(a)$  for  $\bar{e} > 1$

### Conclusion

The analysis of the extensive development path is carried out on the basis of the economic-mathematical model of the enterprise's net proceeds function, which depends on such parameters as price, sales volume, absolute value of the price elasticity of demand, inflation factor, cost factor of production and the level of tax rate. The nature of dependence of net proceeds on these parameters was studied and, based on the obtained results; conclusions were made on the possibility of increasing of net proceeds, as well as on the positive prospects of production and trade. For each of the parameters of the model, not only the conditions for reaching the maximum of net proceeds are indicated, but also the preconditions for its reduction. For completeness of research graphical illustrations of studied dependences of enterprise net proceeds for each of the economic variable model are given.

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