УДК 330.43

ПРОГНОЗИРОВАНИЕ ВОЛАТИЛЬНОСТИ ВАЛЮТНОГО РЫНКА НА БАЗЕ ARCH-МОДЕЛЕЙ (НА ПРИМЕРЕ ПАРЫ EUR/USD)

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Исследовано применение моделей с условной гетероскедастичностью для моделирования волатильности обменного курса EUR/USD для ежедневных наблюдений в период с 1 января 2010 г. по 30 декабря 2016 г. Проанализированы как асимметричные, так и симметричные модели, которые выявили основные особенности валютных котировок, такие как кластеризация волатильности и эффект рычага. Была рассмотрена эффективность прогнозирования волатильности с помощью моделей GARCH и GARCH-M, а также EGARCH, GJR-GARCH и APARC. Изучены остатки данных моделей. Сделан вывод о том, что лучшими моделями прогнозирования волатильности обменных курсов EUR/USD за заданный промежуток времени являются APARCH, GJR-GARCH и модель EGARCH с *t*-распределением Стьюдента.

Ключевые слова: волатильность; валютный рынок; прогнозирование; GARCH; GARCH-M; EGARCH; GJR-GARCH; APARC.

Образец цитирования:
Грецкий Р. Е., Карачун И. А. Прогнозирование волатильности валютного рынка на базе ARCH-моделей (на примере пары EUR/USD) // Журн. Белорус. гос. ун-та. Экономика. 2018. № 1. С. 4–13 (на англ.).

For citation:
Hretski R. E., Karachun I. A. Comparison of ARCH models in forecasting volatility on the EUR/USD currency market. *J. Belarus. State Univ. Econ.* 2018. No. 1. P. 4–13.

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Journal of the Belarusian State University. Economics. 2018. No. 1. P. 4–13

COMPARISON OF ARCH MODELS IN FORECASTING VOLATILITY (ON THE EUR/USD CURRENCY MARKET)

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In this paper the generalized autoregressive conditional heteroscedastic models were applied for modeling volatility of the exchange rate of EUR/USD for daily observations using dataset of period starting 1 January 2010 to 30 December 2016. The paper analyzes both asymmetric and symmetric models that found numerous facts about exchange rate returns such as volatility clustering and leverage effect. The performance of GARCH and GARCH-M models as well EGARCH, GJR-GARCH and APARCH (models with different residual distributions were analyzed to a given dataset. The best models for forecasting volatility of EUR/USD exchange rates are APARCH, GJR-GARCH and EGARCH model with Student's *t*-distribution.

Key words: volatility; forex market; forecast; GARCH; GARCH-M; EGARCH; GJR-GARCH; APARC.

Description of existing approaches to definition and analysis of volatility

Financial markets can be analyzed in very different ways. On the one hand, there are economic theories that focus directly on the valuation of financial assets, on the other – theories related to individual markets (currency, interest, stock, derivatives, etc.). Among the well-known examples, we can name the parity of exchange rates, the model of the time structure of interest rates, the capital asset pricing model (CAPM), and the option pricing model of Black–Scholes. Most of these models are based on theoretical concepts that use expectations, utility functions and risk preferences. It is usually assumed that market participants act rationally, have rational expectations and are not prone to risk. Under such conditions, prices and returns can be determined within the framework of equilibrium models, such as CAPM, which «clean» the markets, i. e. equalize supply and aggregate demand. Another approach follows the arbitrage theory (for example, Black–Scholes), suggesting that the possibility of obtaining a risk-free profit will be immediately noticed by market participants and eliminated by adjusting prices [1]. Arbitrage theory and the equilibrium theory are closely related, although the former is repelled by fewer assumptions, and the latter contains more precisely defined solutions for complex situations.

In the last few years the forex market has become the most liquid and volatile among all other financial markets in the whole world. This fact resulted in unpredictable behavior of some of the currency markets, so the dynamics of the foreign exchange market can become even more dangerous in the nearest future. That's why it is essential to study some of the important historical events relating to currencies and currency exchange. The modeling and forecasting exchange rates volatility has important implications in a range of areas in macroeconomics and finance. One of the most popular implications is Value at Risk (VaR)[1] is a risk measurement tool based on loss distributions. The Basel III framework, which was developed by the Basel Committee on Banking Supervision requires that banks and investment firms keep a minimum amount of capital to cover potential losses from their exposure to many kind of risks, including credit risk, operational risk and market risk. For measuring market risk, Basel III recommend the use of VaR. Inaccurate portfolio VaR estimates may lead firms to maintain insufficient risk capital reserves so that they have an inadequate capital cushion to absorb large financial shocks [2].

Numerous models were developed to analyze volatility for different countries, currencies and other financial assets. The most applied models for forecasting exchange rate volatility is the ARCH[2] model, which was written by R. F. Engle in 1982 and the generalized ARCH or GARCH model developed by T. A. Bollerslev and R. Taylor in 1986 [3]. However, in numerous papers GARCH models have been criticized because they do not provide a theoretical explanation of volatility. Another known weakness of the GARCH model is in fact that the model also responds equally to asymmetric shocks, and cannot cope with significantly skewed time series which can often result in biased estimates of the conditional volatility. The key purpose of ARCH

[1] VaR is a statistical technique used to measure and quantify the level of financial risk within a firm or investment portfolio over a specific time frame. This metric is most commonly used by investment and commercial banks to determine the extent and occurrence ratio of potential losses in their institutional portfolios.

[2] ARCH model – autoregressive conditional heteroscedastic model.

model and its variations is to estimate the conditional variance of a given time series. In his early work R. F. Engle described the conditional variance by a quadratic function of lagged values of time series. Than Bollerslev in 1986 had extended the basic Engle's ARCH model and described the conditional variance in a different way: he stated that conditional variance depends on its own lagged values and the square of the lagged values of shocks. To overcome ARCH and GARCH models' drawbacks Bollerslev used the Student's *t*-distributions. Up-to-date there were created tons of variation of different GARCH models or so called «extensions». The most used are Exponential GARCH, Threshold GARCH, GJR-GARCH (Glosten Jagannathan Runkle GARCH) model and power GARCH models. These models were developed address some or all of weaknesses mentioned above. Because of all these studies the modern GARCH family models capture heteroscedasticity and volatility clustering in financial data across different financial markets [3; 4].

Studying the volatility of the return on assets has made an important contribution to understanding modern financial markets. Volatility is considered a measure of risk, and the riskiness of any financial asset is a decisive characteristic that determines its equilibrium price.

The main objective of this paper is to model exchange rate volatility for EUR/USD, by applying different univariate specifications of GARCH type models for daily observations of the exchange rate time series for the period between 1 January 2010 and 30 December 2016. The volatility models applied in this paper include the GARCH (1, 1), GARCH-M (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1), and Power GARCH (1, 1).

The auto regression process and the theoretical foundations of ARCH and GARCH models

ARCH models were developed to account for empirical patterns in financial data. Many financial time series are characterized by the following facts:

- non-stationarity of asset prices with a stationary return (relative indicator);
- autocorrelation in time series of returns is usually absent or extremely low;
- the volatility is clustered into high and low volatility intervals;
- the distribution of financial time series, as a rule, does not have a normal distribution, and distributions are characterized by long tails;
- in most of the time series data effect of the leverage can be observed, which result in fact that changes in the prices of a financial instrument (stocks, quotes, currencies, etc.) negatively correlate with changes in volatility rate;
- volatility of various financial assets within the same financial market very often move together.

The ARCH model was first proposed by R. F. Engle and was based on modeling the standard deviation of the yield of a financial instrument using the sum of constant basic volatility and a linear function of the absolute values of several recent changes in its prices [5]. The level of volatility is calculated by the following recursive formula (according to ARCH (q)):

$$\sigma_t^2 = a + \sum_{i=1}^{q} b_i \varepsilon_{t-i}^2,$$

where a – base volatility (constant); ε – previous change in prices; q – model parameter – the number of recent price changes that affect the current volatility rate; b_i – weight coefficients that determine the degree of influence of previous price changes on the current rate of volatility.

ARCH model assumes the dependence of volatility only on the squares of past values of time series. If we assume that it also depends on the past values of the conditional variance itself, we get a GARCH and other modifications. Their main task is to consider the information asymmetry: bad news (negative shocks) usually have a greater impact on volatility than good news (positive shocks), so volatility is higher in the falling market than on the growing one – the leverage effect. In the framework of classical GARCH models, this effect cannot be explained, since the conditional variance depends on the squares of the past values of the series and does not depend on their signs.

GARCH:

$$\sigma_t^2 = a + \sum_{i=1}^{q} b_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} c_j \sigma_{t-j}^2,$$

EGARCH:

$$\ln \sigma_t^2 = a + \sum_{i=1}^{q} b_i g\left(\varepsilon_{t-i}\right) + \sum_{j=1}^{p} c_j \ln \sigma_{t-j}^2, \quad g\left(\varepsilon_t\right) = \delta_1 \varepsilon_t + \delta_2 \left(\left|\varepsilon_t\right| - \sqrt{\frac{2}{\pi}}\right).$$

ARCH model is only the starting point of an empirical study and relies on a wide range of specification tests. Some practice-oriented discrepancies have been identified relatively recently, for example, the definition and modeling of shocks and the problem of modeling asymmetry [3].

The first studies in the field of econometric modeling of volatility were extremely parametric, but in recent years there has been a shift towards less parametric and even completely nonparametric methods. Non-parametric approaches for modeling volatility, which, as a rule, do not make assumptions about functional distributions, allow to obtain flexible and at the same time steady estimates of actual volatility.

Methodology behind analyzing EUR/USD volatility

At its most basic level, fitting ARIMA and GARCH models is an exercise in uncovering the way in which observations, noise and variance in a time series affect subsequent values of the time series. Such a model, properly fitted, would have some predictive utility, assuming of course that the model remained a good fit for the underlying process for some time in the future.

In finance, high risk is often expected to lead to high returns. To model such a phenomenon one may consider the GARCH-M Model of Engle, Lilien, and Robins developed in 1987 where «M» stands for GARCH in the mean. This model is an extension of the basic GARCH framework which allows the conditional mean of a sequence to depend on its conditional variance or standard deviation. A simple GARCH-M (1, 1) model is given by

$$r_t = \mu + \lambda\sigma_t^2 + y_t,\ y_t = \sigma_t\varepsilon_t,\ \varepsilon_t \quad N\left(0, \sigma_t^2\right),$$

$$\sigma_t^2 = \omega + \alpha_1 y_{t-1}^2 + \beta_1\sigma_{t-1}^2,$$

where μ and λ are constants. The parameter λ is called the risk premium parameter. A positive λ indicates that the return is positively related to its volatility.

In practice, the price of financial assets often reacts more pronouncedly to «bad» news than «good» news. Such a phenomenon leads to a so-called leverage effect, as first noted by Black in 1976. The term «leverage» stems from the empirical observation that the volatility (conditional variance) of a stock tends to increase when its returns are negative. The leverage effect causes the asymmetries of variance dynamics and points out the drawbacks of GARCH model because of its symmetric effect towards the conditional variance. In order to capture the asymmetry in return volatility («leverage effect»), a new class of models was developed, termed the asymmetric GARCH models. This paper uses the following asymmetric GARCH models; EGARCH GJR-GARCH and Asymmetric Power ARCH (APARCH) model for capturing the asymmetric phenomena [5; 6].

Validation and comparison of different ARCH models

Conditional volatility is usually estimated using different probability distributions. These distributions can be found and estimated via «rugarch» package in R and Python. This package includes normal, Student *t* and skewed Student *t*-distribution. One of Engle's key assumptions is that asset returns follow a normal distribution. However, in practice the asset returns (as well as fluctuations on currency markets) are not normally distributed, so the normality assumption could cause significant bias in VaR estimation and could underestimate the volatility. It is indicated that standard GARCH models with normal empirical distributions have inferior forecasting performance compared to models that reflect skewness and kurtosis in innovations. To capture the excess kurtosis in financial asset returns, Bollerslev in 1987 introduced the GARCH model with a standardized Student's *t*-distribution with 2 degrees of freedom. The common methodology used for GARCH estimation is maximum likelihood [3; 4]. The parameters of the GARCH model can be found by maximizing the objective log-likelihood function:

$$\ln L(\theta) = -\frac{1}{2}\sum_{t=1}^{n}\left[\ln(2\pi) + \ln\left(\sigma_t^2(\theta)\right) + z_t^2(\theta)\right],$$

where θ is the vector of parameters (ω, μ, α_i, β_j) estimated that maximize the objective function (lnθ); z_t represents the standardized residual.

Maximum likelihood estimates of the parameters are obtained by numerical maximization of the log-likelihood function using the Marquardt algorithm[3]. The quasi-maximum likelihood estimator (QMLE) used since then.

Empirical results of forecasting EUR/USD forex market

The data set consists of the daily currency exchange rate of the EUR vs. USD (EUR/USD). These data are obtained from European Central Bank (ECB) website (*www.ecb.europa.eu*). The data set was for the period

[3]In mathematics and computing the Levenberg – Marquardt algorithm (LMA), also known as the damped least-squares (DLS) method, is used to solve non-linear least squares problems. These minimization problems arise especially in least squares curve fitting.

from 1 January 2010 to 30 December 2016, a total of 1826 observation. A visual inspection of fig. 1 shows that daily EUR/USD exchange rate prices are not stationary. To test for stationarity an Augmented Dickey – Fuller test (ADF) for a unit root in a time series sample is performed. The computed ADF test-statistic in table 1 is (–3.0) which greater than the critical values at 1 % significance level. Therefore, we fail to reject the null hypothesis that there is a unit root and that the series needs to be differenced to make it stationary.

However, if we make a logarithmic transformation of the time series, then we get directly a series of returns of the chosen financial instrument, graphically this process (fig. 2) is very similar to white noise[4].

A plot of the log returns series for EUR/USD exchange rates given in fig. 2 shows periods of high volatility, occasional extreme movements and volatility clustering, as upward movements tend to be followed by other upward movements and downward movements also followed by other downward movements. This indicates that the logarithm of EUR/USD exchange rates is stationary after taking the first-difference, and the ADF test results in table 1 confirm the stationarity of the return series data. The computed ADF test-statistic in table 1 is (−9.3) which smaller than the critical values at 5 % significance level.

The mean of analyzed data is positive, suggesting that exchange returns increase slightly over time. The coefficient of skewness indicates that returns have asymmetric distribution, i. e., they are skewed to the left. The kurtosis of returns is 73.6776 which is greater than three, indicating that the distribution of returns follows a fat-tailed distribution, thereby exhibiting one of the important characteristics of financial time series data, namely that of leptokurtosis. The non-normality condition is supported by a Jarque – Bera test which indicate that the null hypothesis of normality is rejected at the 5 % level of significance.

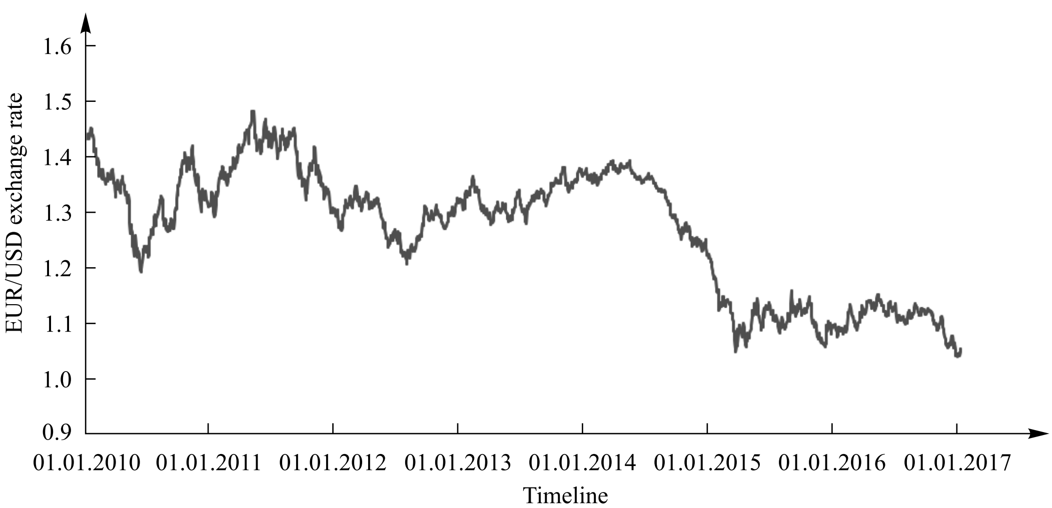

Fig. 1. Daily EUR/USD currency exchange rates

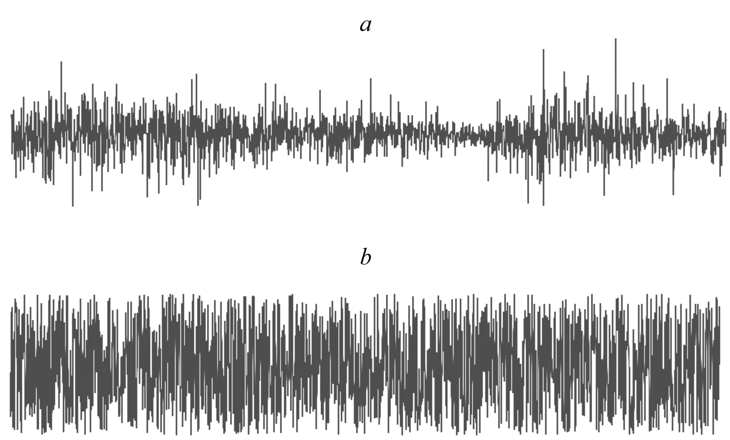

Fig. 2. Logarithmic price indices (*a*) and a typical Gaussian white noise (*b*) for the EUR/USD currency quote

[4]A scalar random process is called white noise if it is stationary (in a broad sense) and has a constant spectral density, called the intensity of white noise.

Table 1

Augmented Dickey – Fuller test of the daily returns

ADF test statistic	Confidence level, %	Critical Value
–9.3	1	–3.436 1
	5	–2.863 2
	10	–2.567 7

The Ljung – Box test is applied to the daily log returns of the EUR/USD exchange rates and the test results are shown in table 2. The null hypothesis of the Ljung – Box is rejected for the returns, squared returns and absolute returns, at lags 1, 6, 10, 15 and 20. The test statistics are statistically significant with *p*-values not greater than 0.01, indicating that the returns are not white noise. Indeed, the daily exchange rate returns exhibits correlation.

Table 2

***p*-Values based on the Ljung – Box test for the EUR/USD exchange rates**

Variable	Metric	Lag 1	Lag 6	Lag 10	Lag 15	Lag 20
Returns	Qm *p*-value	10 (0.000 3)	40 (0.000 0)	50 (0.000 0)	60 (0.000 0)	90 (0.000 0)
Squared returns	Qm *p*-value	6 (0.02)	20 (0.01)	30 (0.000 4)	40 (0.002)	600 (0.000 0)
Absolute returns	Qm *p*-value	200 (0.000 0)	800 (0.000 0)	1000 (0.000 0)	1200 (0.000 0)	1400 (0.000 0)

From the results of Ljung – Box test in table 2 and the autocorrelation (ACF) and partial autocorrelation (PACF) for the exchange rate return series, absolute and squared return series shows that the return series exhibit autocorrelation at some lags at 5 % level of significance. The presence of autocorrelation detected in the log return can be removed by fitting the simplest plausible ARMA (p, q) model to the data. On the other hand, the autocorrelation detected in the squared log returns, indicate that there exists conditional heteroskedasticity of the exchange rate returns series which could be removed by fitting the simplest plausible GARCH model to the ARMA filtered data.

An ARMA (p, q) model is used to fit the mean returns, as it provides a flexible and parsimonious approximation to conditional mean dynamics. The Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) are used to determine the order of ARMA (p, q) models. The ACF and PACF plots given in table 3 suggest that the returns may be modeled by an ARMA (2, 2) process. It is often proposed to use extended autocorrelation function (EACF) technique to identify the orders of a stationary or non-stationary ARMA process based on iterated least square estimates of the autoregressive parameters. The output of EACF is a two-way table, where the rows correspond to AR order p and the columns to MA order q. Therefore, the EACF suggests that the daily log returns of EUR/USD exchange rate follow an ARMA (2, 0) model. This agrees with the result in table 3 suggested by the best fitting model selected based on Bayesian Information Criterion (BIC) values. The criterion is to choose a model with minimum AIC and BIC and largest log-likelihood function. BIC always gives penalty for the additional parameters more than AIC does. So the ARMA (1, 1) is selected as the mean equation that mainly takes account of the BIC.

Table 3

Criterion for ARMA (*p*, *q*) order selection

Model specification	BIC value	AIC value
ARMA (1, 0)	–19 267	–19 349
ARMA (1, 1)	**–19 152**	**–19 234**
ARMA (1, 2)	–19 326	–19 326
ARMA (1, 3)	–19 337	–19 337
ARMA (2, 0)	–19 348	–19 348
ARMA (2, 1)	–19 359	–19 359

Ending table 3

Model specification	BIC value	AIC value
ARMA (2, 2)	–19 370	–19 370
ARMA (2, 3)	–19 381	–19 381
ARMA (0, 0)	–19 392	–19 392
ARMA (0, 1)	–19 403	–19 403
ARMA (0, 2)	–19 370	–19 370
ARMA (0, 3)	–19 381	–19 381
ARMA (0, 0)	–19 348	–19 348

The results of the fitted ARMA (1, 1) – GARCH (1, 1) and ARMA (1, 1) – GARCH-M (1, 1) models to the EUR/USD log return series with normal distribution, Student's *t*-distribution and skewed *t*-distribution for the standardized residuals are presented in table 4. The estimates of the model parameters are all significant for normal, Student's *t* and skewed *t* distribution except for the ω parameter which is not significant for all the distributions. The estimates of φ_1 and φ_2 are significant, supporting the use of the ARMA (1, 1) model for the returns. Volatility shocks are persistent since the sum of the ARCH and GARCH coefficients are very close to one. The Box – Pierce *Q*-statistics is insignificant up to lag 20, indicating that there is no excessive autocorrelation left in the residuals. Comparing the log-likelihood and information criterion in table 4 within the three conditional distributions, the model with conditional distribution of skewed *t* has larger log-likelihood and smaller information criterion statistics than estimated by normal and *t*-distribution that means this model is better fitted.

Table 4

Estimation of ARMA (1, 1) – GARCH (1, 1) and ARMA (1, 1) – GARCH-M (1, 1) with different distributions[5]

Coefficient	ARMA (1, 1) – GARCH (1, 1)			ARMA (1, 1) – GARCH-M (1, 1)		
	Normal	*t*	Skew *t*	Normal	*t*	Skew *t*
μ	0.000 16	0.000 11	0.000 13	0.000 17	0.000 98	0.000 13
	–0.003 01	–0.016 13	–0.016 44	–0.004 33	–0.018 12	–0.016 13
AR (1)	0.125 26	0.128 23	0.128 03	0.121 23	0.128 15	0.127 84
	0	0	0	0	0	0
MA (1)	–0.042 26	–0.032 34	–0.031 26	–0.043 27	–0.031 31	–0.033 37
	0	–0.010 54	–0.011 31	–0.000 01	–0.011 26	–0.010 31
Omega	0	0	0	0	0	0
	–0.637 38	–0.434 63	–0.462 83	–0.653 27	–0.461 72	–0.463 72
α	0.110 27	0.310 93	0.299 24	0.109 15	0.312 62	0.309 33
	0	0	0	0	0	0
β	0.782 72	0.572 32	0.573 39	0.791 27	0.572 44	0.573 91
	0	0	0	0	0	0
Skew	–	–	–	–	–	1.021 79
						0
Shape	–	3.193 37	1.002 19	–	3.207 02	3.209 23
		0	0		0	0
LLF	11 318	11 916	11 921	11 318	11 916	11 921
AIC	–8.116 6	–8.377 8	–8.377 2	–8.116 6	–8.377 8	–8.377 2
BIC	–8.103 9	–8.363	–8.830 3	–8.103 9	–8.363	–8.830 3

[5] *p*-Values are shown in parentheses.

To capture the asymmetry dynamics and the presence of the «leverage effect» in the EUR/USD exchange rate returns, the nonlinear asymmetric models; ARMA (1, 1) – EGARCH (1, 1), ARMA (1, 1) – GJR-GARCH (1, 1) and ARMA (1, 1) – APARCH (1, 1) with conditional distributions; normal distribution, Student's *t* distribution and skewed *t*-distribution are fitted to the exchange returns. Table 5 gives the results of the parameter estimates for the ARMA (1, 1) – EGARCH (1, 1), ARMA (1, 1) – GJR-GARCH (1, 1) and ARMA (1, 1) – APARCH (1, 1) models. The parameters estimate for these three models are all significant except for the mean under the ARMA (1, 1) – EGARCH (1, 1) for the normal and skew *t*-distribution, also the coefficient of the second term of autoregressive process under the skew *t*-distribution and the coefficients of α_1 under the Student's *t* and skew *t*-distribution are not significant. For both the ARMA (1, 1) – GJR-GARCH (1, 1) and ARMA (1, 1) – APARCH (1, 1) ω is not significant for all the distribution. The parameter γ is not significant for the ARMA (1, 1) – APARCH (1, 1) under the *t*-distribution. The coefficient γ in the case of ARMA (1, 1) – APARCH (1, 1) is statistically significant at level of significance of 5 % implying that there is an asymmetry under the normal distribution. On the other hand, its negative value indicates the presence of the «leverage effect». The coefficient γ in the ARMA (1, 1) – EGARCH (1, 1) and ARMA (1, 1) – GJR-GARCH (1, 1) is significantly different from zero, which indicates the presence of asymmetry. The value of γ which is less than zero implies presence of the «leverage effect». According to the log-likelihood value and information criterion of the estimated models, the APARCH model has the larger log-likelihood value and smaller information criterion compared with EGARCH model and GJR-GARCH model. Secondly, comparing within the APARCH models under normal distribution, and Student's *t*-distribution, the model with conditional Student's *t*-distribution outperforms the normal distribution that means this model is superior in modeling the EUR/USD exchange rate returns with asymmetry and fat tail.

The estimated power parameter δ in the APARCH model is 2.44 that is slightly different from the estimated result of Ding, Granger and Engle work under the normal distribution that is 1.43. This may be caused by the time period of the data is different and then mean equation is different to model the data. However, δ in this paper is still significantly different from 1 (GJR-GARCH) or 2 (GARCH). When the conditional distribution changes to *t*-distribution δ is getting smaller to 0.73. However, using the same test as in Ding, Granger and Engle's paper: let l_0 be the log-likelihood of value under the GARCH model which is set as the null hypothesis, while the alternative hypothesis is APARCH model with log-likelihood is l, then $2(l - l_0)$ have a χ^2 distribution with 2 degrees of freedom when H_0 is true. Then, under the Student's *t*-distribution $2(l - l_0) = 2(12\ 547 - 12\ 511) = 72$, which means we can reject the null hypothesis that the data is generated from GARCH model. In the same way, we can reject that the data is generated from EGARCH model and GJR-GARCH model.

Table 5

Estimation of ARMA (1, 1) – EGARCH (1, 1) and ARMA (1, 1) – GJR-GARCH (1, 1) with different distributions

Coefficient	ARMA (1, 1) – EGARCH (1, 1)			ARMA (1, 1) – GJR–GARCH (1, 1)			ARMA (1, 1) – APARCH (1, 1)	
	Normal	*t*	Skew *t*	Normal	*t*	Skew *t*	Normal	*t*
μ	–0.000 128 (0.100 463)	0.000 069 (0.011 437)	0.000 074 (0.051 372)	0.000 158 (0.000 428)	0.000 141 (0.011 327)	0.000 094 (0.010 423)	0.000 158 (0.029 627)	0.000 052 (0.000 00)
AR (1)	0.125 26 (0.000 01)	0.128 233 (0.000 00)	0.128 031 (0.000 00)	0.135 163 (0.000 00)	0.138 173 (0.000 00)	0.128 072 (0.000 00)	0.133 617 (0.000 00)	0.096 342 (0.000 00)
MA (1)	–0.042 26 (0.000 00)	–0.028 34 (0.031 78)	–0.028 84 (0.062 82)	–0.089 21 (0.000 01)	–0.043 34 (0.012 63)	–0.043 77 (0.012 37)	–0.109 36 (0.000 00)	–0.012 56 (0.000 00)
Omega	–0.523 262 (0.000 00)	–0.562 862 (0.000 01)	–0.562 984 (0.000 01)	0 (0.656 32)	0 (0.496 23)	0 (0.503 261)	0 (0.893 57)	0 (0.883 32)
α	0.111 272 (0.000 00)	0.036 272 (0.443 167)	0.036 452 (0.441 858)	0.135 367 (0.000 00)	0.336 281 (0.000 00)	0.336 473 (0.000 00)	0.095 427 (0.000 00)	0.372 461 (0.000 00)
β	0.893 273 (0.000 00)	0.898 471 (0.000 00)	0.898 449 (0.000 00)	0.848 617 (0.000 00)	0.672 472 (0.000 00)	0.673 169 (0.000 00)	0.848 558 (0.000 00)	0.838 175 (0.000 00)
Gamma	0.484 044 (0.000 00)	0.554 24 (0.000 00)	0.557 42 (0.000 00)	–0.023 56 (0.026 716)	–0.083 16 (0.049 136)	–0.083 77 (0.042 325)	–0.051 262 (0.036 282 3)	–0.083 261 (0.065 42)
Delta	–	–	–	–	–	–	2.432 16 (0.000 00)	0.712 335 3 (0.000 00)

Ending table 5

Coefficient	ARMA (1, 1) – EGARCH (1, 1)			ARMA (1, 1) – GJR–GARCH (1, 1)			ARMA (1, 1) – APARCH (1, 1)	
	Normal	*t*	Skew *t*	Normal	*t*	Skew *t*	Normal	*t*
Skew	–	–	1.002 189 (0.000 00)	–	–	1.002 189 (0.000 00)	–	–
Shape	–	2.373 67 (0.000 00)	2.373 975 (0.000 00)	–	3.193 371 (0.000 00)	3.209 371 (0.000 00)	–	2.103 16 (0.000 00)
LLF	11 557	12 405	12 405	11 904	12 413	12 414	10 997	12 447
AIC	–8.200 4	–8.816 6	–8.816 3	–8.523 4	–8.818	–8.817 6	–8.516 7	–8.901 4
BIC	–8.162 6	–9.803 5	–8.803 1	–8.512 9	–8.809 3	–8.805 3	–8.503 16	–8.893 61

The GARCH models with the innovations of Student's and skewed Student's *t*-distributions have a better fit in general than the models with normal distribution innovations since they have the highest log-likelihood function (LLF) and smallest AIC and BIC. Secondly, the values of the AIC, BIC and LLF for all the models with Student's and skewed Student's *t*-distributions innovations are not significantly different. This implies that the models with Student's *t* and skewed Student's *t*-distributions innovations would result in the same conclusions.

The figures in table 4 shows that model ARMA (1, 1) – APARCH (1, 1) is well specified. The ACF of the square standardized residuals compares well with the ACF of the square returns. This shows that ARMA (1, 1) – APARCH (1, 1) Student *t*-model sufficiently explains the heteroscedasticity effect in the returns, thus we can conclude that the model fit the EUR/USD returns well. The Ljung – Box test of the standardized residuals at different lags confirms that standardized residuals have no correlation.

Results of modelling EUR/USD conditional volatility

Modeling and forecasting the volatility of exchange rate returns has become an important field of empirical research in finance. This is because volatility is considered as an important concept in many economic and financial applications like asset pricing, risk management and portfolio allocation [7]. This paper attempts to explore the comparative performance of different econometric volatility forecasting models in the terms of their ability to estimate VaR in the EUR/USD exchange rates. Five different models were considered in this study. The volatility of the EUR/USD returns have been modeled by using a univariate GARCH models including both symmetric and asymmetric models. That captures most common stylized facts about exchange returns such as volatility clustering and leverage effect. These models are GARCH (1, 1), GARCH-M (1, 1), exponential GARCH (1, 1), GJR GARCH (1, 1) and APARCH (1, 1) following three residual distributions namely: normal, Student's *t*-distribution and Skewed Student's *t*-distribution. The first two models are used for capturing the symmetry effect whereas the second group of models is for capturing the asymmetric effect. The study used the EUR/USD exchange rates data from the European Central Bank (ECB) for the period 1 January 2010 to 30 December 2016. Based on the empirical results presented, the following can be concluded.

There is strong evidence that the above-mentioned models could characterize daily returns. The EUR/USD data showed a significant departure from normality and existence of conditional heteroscedasticity in the residuals series. Descriptive statistics for the EUR/USD exchange rates show presence of negative skewness and excess kurtosis. The results of the conducted ARCH-LM test point out significant presence of ARCH effect in the residuals as well as volatility clustering effect. Standardized residuals and standardized residuals squared were white noise. The empirical results have indicated that the most adequate GARCH models for estimating and forecasting VaR in the EUR/USD exchange rates are the asymmetric APARCH, GJR-GARCH and EGARCH model with Student's *t*-distribution. These models have a better fit of the exchange returns, since they have the largest log-likelihood function and smallest AIC and BIC. The findings have important implications regarding VaR estimation in volatile times, market timing, portfolio selection etc. that have to be addressed by investors and other risk managers operating in emerging markets. However, the limitation of the study is that the empirical research focused only on the EUR/USD exchange rate and therefore the findings cannot be generalized to other exchange rates in the market. In the future research a wider sample of exchange rates should be used to compare the performance of the most commonly used foreign currencies in the market (also in relation to BYN currency) and the inclusion of other asymmetric GARCH-type models, testing and comparing their predictive performance.

Библиографические ссылки

1. *Black F.* The Pricing of Options and Corporate Liabilities // J. Political Econ. 1973. № 81. P. 637–654.
2. *Black F.* Studies of Stock Price Volatility Changes // Proceedings of the 1976 Meeting of the Business and Economic Statistics Section. Washington DC, 1976. P. 177–181.
3. *Bollerslev T.* A Conditional Heteroskedasticity Time Series Model for Speculative Prices and Rates of Return // Rev. Econ. Stat. 1987. № 69 (3). P. 542–547.
4. *Bollerslev T.* Generalized Autoregressive Conditional Heteroscedasticity // J. Econom. 1986. No. 31 (3). P. 307–327.
5. *Engle R. F.* Autoregressive Conditional Heteroskedasticity with the Estimates of the Variance of U.K. Inflation // Econometrica. 1982. № 50. P. 987–1008.
6. *Engle R. F., David M., Russel P.* Estimating time varying risk premia in the term structure: the ARCH-M model // Econometrica. 1987. No. 55. P. 391–407.
7. *Грецкий Р. Е., Карачун И. А.* Применение *logit*-модели для прогнозирования волатильности валютного курса // Экономика, моделирование, прогнозирование : сб. науч. тр. Минск, 2016. № 10. С. 21–28.

References

1. Black F. The Pricing of Options and Corporate Liabilities. *J. Political Econ.* 1973. No. 81. P. 637–654.
2. Black F. Studies of Stock Price Volatility Changes. *Proceedings of the 1976 Meeting of the Business and Economic Statistics Section.* Washington DC, 1976. P. 177–181.
3. Bollerslev T. A Conditional Heteroskedasticity Time Series Model for Speculative Prices and Rates of Return. *Rev. Econ. Stat.* 1987. No. 69 (3). P. 542–547.
4. Bollerslev T. Generalized Autoregressive Conditional Heteroscedasticity. *J. Econom.* 1986. No. 31 (3). P. 307–327.
5. Engle R. F. Autoregressive Conditional Heteroskedasticity with the Estimates of the Variance of U.K. Inflation. *Econometrica.* 1982. No. 50. P. 987–1008.
6. Engle R. F., David M., Russel P. Estimating time varying risk premia in the term structure: the ARCH-M model. *Econometrica.* 1987. No. 55. P. 391–407.
7. Gretskiy R. E., Karachun I. A. [Implementation of logit model in volatility forecasting of exchange rate]. *Ekonomika, modelirovanie, prognozirovaniye* [Economics, modelling, forecasting] : collect. of sci. works. Minsk, 2016. No. 10. P. 21–28.

Received by editorial board 18.01.2018.