C. Математические и количественные методы
C. Mathematical and Quantitative Methods

О ВОЗМОЖНОСТИ ГОСУДАРСТВЕННОГО РЕГУЛИРОВАНИЯ ПРИ ЭКСТЕНСИВНОМ ПУТИ РАЗВИТИЯ ПРОИЗВОДСТВА

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Исследуются возможности государственного регулирования объема выпуска продукции предпринимателя при расширенном производстве. Предполагается, что государство применяет льготные налоговые ставки в качестве основных стимулов. За основу взята математическая модель выручки предприятия. По результатам исследования модели выявлены условия максимизации дохода предпринимателя: предприятие достигает максимальных выгод при наращивании производства в случаях минимальной льготной налоговой ставки, максимальных значениях темпа роста цены и коэффициента издержек от производства дополнительной единицы продукции. Установлено также, что максимизации дохода способствует функционирование предприятия либо на рынках товаров с высокой ценовой эластичностью, либо на рынках товаров с ценовой эластичностью, близкой к нулю.

Ключевые слова: государственное регулирование; доход; льготные условия; налоговая ставка; наращивание выпуска.

ON THE POSSIBILITY OF STATE REGULATION IN THE EXTENSIVE PATH OF DEVELOPMENT OF PRODUCTION

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The paper explores the possibilities of state regulation of the volume of the entrepreneur's output in expanded production. It is assumed that the state applies preferential tax rates as the main incentives. The mathematical model of the company's proceeds is the foundation of the study. Based on the results of the model study, the conditions for maximizing the entrepreneur's income are revealed: the enterprise achieves maximum benefits in the conditions of increasing production in cases of the minimum preferential tax rate, the maximum values of the rate of price growth and the coefficient of costs in the production of an additional unit of products. The research also found that the maximization of income arises when the enterprise operates in the markets of goods with high price elasticity or in the markets of goods with price elasticity close to zero.

Keywords: state regulation; income; preferential terms; tax rate; build-up of output.

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Introduction

This article is a continuation of the research carried out in [1–4].

The mathematical model of the proceeds of an enterprise which uses the extensive way of development of production and trade was first analyzed in detail in the article [3]. It was found out that the company's proceeds depend on such indicators as the coefficient of sales increase, the level of inflation, the coefficient of costs in the production of an additional unit of products and the coefficient of price elasticity of demand. The influence of each of the parameters on the difference between the initial proceeds of the entrepreneur and the proceeds, when he enters the extensive development path, was also considered. In particular, it was identified that the maximum benefit from the transition to an increase in production is achieved under the following conditions: when the company operates in the markets of highly elastic goods, has a small coefficient of costs and a small coefficient of sales increase, operates with a minimum level of inflation.

In [1, 2] the dependences of the maximum income of an enterprise, that follows the extensive path of development, on various market parameters were studied. As a result, it was revealed that an entrepreneur maximizes his income in the event of an increase in production, if he is operating in the markets of highly elastic goods, while the share of total tax payments from the proceeds and the coefficient of costs in the production of an additional unit of products tend to zero.

In [4] the qualitative characteristics of the dependencies obtained in [3] were analyzed, i. e. the indicators of proceeds elasticity were analyzed. The values for each parameter of the proceeds model were determined, with a tendency toward which the proceeds are the most elastic. In addition, several bifurcation points were revealed, reflecting the fact that with a small change in their values, the sign of the proceeds elasticity function changes.

However, in papers [1–4] such an important issue from an economic point of view as the possibility of state regulation of an entrepreneur's income wasn't considered. The proposed article is devoted to this problem of the theory of market economics. It is formulated and studied in the following formulation.

It is assumed that in a certain period of time the state plans to expand the production of some products. Therefore, in order to encourage the producer to increase the output, the government can appoint certain benefits for expanded production. So the parameter of the assigned tax benefits gives the potential to regulate the income of the enterprise. We study the theoretical aspects of one of the forms of such actions.

Proceeds

We recall the definitions and notation of [1; 3]. Let the producer sells q pieces of goods at a price p for one unit on the market for a certain period of time. Then the proceeds from the sale will be qp of monetary units.

Suppose that in order to increase proceeds, the entrepreneur increases production and sales of his products to the value $q_1 = q + \Delta q$, $\Delta q > 0$. Investigations of such case are carried out under the following assumptions.

1. *At the end of the observed period of time there is a natural inflation in the amount of* $\sigma(\sigma > 0)$ *monetary units per unit of output. It is called **underlying inflation** for the period of time associated with external factors (rising prices for energy resources, raw materials, transportation costs, additional services, etc.).*

2. *Without underlying inflation, the increase in output in the market by an amount Δq in accordance with the law of demand leads to a price change to a certain value p_1, where $p_1 \leq p$, and taking into account underlying inflation – to the value $p_1 + \sigma$ per unit of goods.*

3. *Underlying inflation does not cause changes in the volume of sales (the government reduces the impact of inflation on consumer demand, for example, using indexation of wages, etc.).*

Next let's study the possibility of benefit from increasing production of products by the entrepreneur and investigate his risk of possible losses and miscalculations.

Let a $(0 < a < 1)$ denotes ***the coefficient of costs in the production of an additional unit of products*** for each of Δq units. Consequently, additional products add at the end of the observed period the amount $\Delta q(p_1 + \sigma)(1 - a)$ to the proceeds $q(p_1 + \sigma)$.

Thus, the total proceeds are represented by the formula

$$R = q(p_1 + \sigma) + \Delta q(p_1 + \sigma)(1 - a). \tag{1}$$

We transform the right side of (1) using the following considerations. In the paper [5, p. 24] the formula (1.17) for the sales volume $q_1 = q_1(p)$ is given as a function of the price p. It is obtained by expanding this function in a Taylor series. We use this formula in the first approximation, assuming that p is the initial value. Then we can write:

$$q_1 = q\left(1 - \frac{e(p_1 - p)}{p}\right), \tag{2}$$

where e is the absolute value of ***the coefficient of price elasticity of demand***.

Using the formula (2), we can define a new price p_1 that was formed at the end of the period by solving this equation with respect to the parameter p_1. Using equality $q_1 - q = \Delta q$ and not taking into account underlying inflation, from the formula (2) we get:

$$p_1 = p\left(1 - \frac{q_1 - q}{eq}\right) = p\left(1 - \frac{\Delta q}{eq}\right). \tag{3}$$

In this case, the proceeds R after the substitution of (3) into (1) take the form:

$$R = q\left(p + \sigma - p\frac{\Delta q}{eq}\right) + A\Delta q\left(p + \sigma - p\frac{\Delta q}{eq}\right), \tag{4}$$

where for brevity of the analysis is laid $A = 1 - a, 0 < A < 1$.

For the convenience of investigation of the properties of the dependence of the proceeds on the parameters of the model, we introduce the following quantities:

$$k_q = \frac{\Delta q}{q}, \ \ k_p = \frac{\sigma}{p}. \tag{5}$$

Let's call them, respectively, ***the coefficient of sales increase*** and ***the level of inflation*** [6; 7], so that k_q 100 % is the percentage increase in sales, and k_p100 % − the percentage of underlying inflation.

As we will see below, in all theoretical arguments there is not always the coefficient k_p, but the quantity $1 + k_p$. Therefore, in order to reduce technical calculations, it is also convenient to use the concept of ***the rate of price growth*** K_p [6], which corresponds to formula:

$$K_p = \frac{p + \sigma}{p} = 1 + \frac{\sigma}{p} = 1 + k_p. \tag{6}$$

Taking into account notations (5) and (6), we transform the right side of the proceeds formula (4) as follows. We take the factor q from the first bracket of the formula (4), and the factor p − from the second bracket. Then finally this formula takes the form:

$$R = qp\left(K_p - \frac{k_q}{e}\right) + qpAk_q\left(K_p - \frac{k_q}{e}\right). \tag{7}$$

It expresses the dependence of the proceeds on the parameters k_q, e, K_p and $a = 1 - A, 0 < A < 1$, which define the mathematical model for research.

Preferential terms

Let us note that the first component of the right side of the formula (7), which equal to $qp\left(K_p - \frac{k_q}{e}\right)$, corresponds to the initial output of the product. Let ***the tax rate*** for it is equal to $i\left(0 < i < 1\right)$. The second component of the right side of the formula (7) is the output of additional units of production. We assign for the second component a special ***preferential tax rate*** j, $0 < j < i$. Then, in accordance with the formula (7), the total income of the producer (denote it as C) can be written as follows:

$$C = qp\frac{\left(eK_p - k_q\right)\left(1 - i\right)}{e} + \frac{qpAk_q\left(eK_p - k_q\right)\left(1 - j\right)}{e} = \frac{qp\left(eK_p - k_q\right)\left(1 - i + A\left(1 - j\right)k_q\right)}{e}.$$

It should be noted that the use of tax incentives in the policy of state regulation of expanded production should be based on the choice of the tax rate j in such a way, that, on the one hand, it stimulates an increase in output, and, on the other hand, enables the producer to profit from this.

Let's analyze these phenomena analytically, using a mathematical model in such a way as to ensure the simultaneous implementation of these goals.

Since the producer's income expressed by the formula above is not assumed to be negative, the first bracket must be positive, i. e.

$$eK_p > k_q. \tag{8}$$

Next, we write down the condition of the producer's interest in increasing the output. In other words, the entrepreneur's income in expanded production should be greater than the initial. Since the initial income is equal to $q\left(p + \sigma\right) = qpK_p$, then the required condition is ensured by inequality

$$C = qp \frac{\left(eK_p - k_q\right)\left(1 - i + A(1-j)k_q\right)}{e} > qpK_p\left(1-i\right). \tag{9}$$

We successively transform this inequality, taking into account the constraint (8). We have:

$$\left(eK_p - k_q\right)\left(1 - i + k_q A(1-j)\right) > eK_p\left(1-i\right) \Leftrightarrow 1 - j > \frac{(1-i)}{A\left(eK_p - k_q\right)}.$$

Taking into account the economic sense of the tax rate j we obtain:

$$0 < j < 1 - \frac{1-i}{A\left(eK_p - k_q\right)}, \quad eK_p > k_q. \tag{10}$$

Let us point out the relationship between this condition and the important economic limitations of the model.

First of all, we define the necessary condition under which the right side of (10) is positive. This, in turn, should provide the opportunity to choose the tax rate j. To do this, we use the inequality

$$1 - \frac{1-i}{A\left(eK_p - k_q\right)} > 0 \Leftrightarrow \frac{A\left(eK_p - k_q\right) - (1-i)}{A\left(eK_p - k_q\right)} > 0.$$

Taking into account the requirement (8), this is equivalent to the system of inequalities

$$A\left(eK_p - k_q\right) > 1 - i, \quad eK_p > k_q.$$

In addition, we must not forget about the condition $0 < j < i$ for a preferential tax rate. Combining all of the above, we come to the conclusion that if the parameters of the model satisfy the system of inequalities:

$$\begin{cases} A\left(eK_p - k_q\right) > 1 - i, \\ j < 1 - \dfrac{1-i}{A\left(eK_p - k_q\right)}, \\ 0 < j < i, \end{cases} \tag{11}$$

then the government plan for stimulating the producer through a preferential tax rate j can be realized.

Income optimization

Let's calculate the maximum income of the entrepreneur depending on the control parameter k_q. For this we consider the income function

$$C\left(k_q\right) = qp \frac{\left(eK_p - k_q\right)\left(1 - i + A(1-j)k_q\right)}{e} \tag{12}$$

defined by (9). This is a quadratic function, the graph of which is the part of the parabola with downward branches. We first indicate the domain of its definition, taking into account the requirements of (11). We note that the variable k_q enters only in the first two inequalities of system (11), so we can use the following equivalent conditions:

$$\begin{cases} A\left(eK_p - k_q\right) > 1 - i, \\ j < 1 - \dfrac{1-i}{A\left(eK_p - k_q\right)} \end{cases} \Leftrightarrow \begin{cases} k_q < eK_p - \dfrac{1-i}{A}, \\ k_q < eK_p - \dfrac{1-i}{A(1-j)}. \end{cases}$$

Since $0 < 1 - j < 1$, then the first inequality from the obtained system can be omitted, because it is a consequence of the second inequality. Consequently, the domain of definition of the function (12) with respect to a variable k_q is given by the inequalities:

$$0 < k_q < eK_p - \frac{1-i}{A(1-j)} \quad \text{for } 0 < j < i. \tag{13}$$

We should note that the function $C(k_q)$ is zero at points $k_q = eK_p$ and $k_q = \dfrac{-(1-i)}{A(1-j)}$. Therefore, the coordinate of the top of the parabola $k_q = \tilde{k}_q$ is the average of the roots of the quadratic function

$$\tilde{k}_q = \frac{1}{2}\left(eK_p - \frac{1-i}{A(1-j)}\right). \tag{14}$$

According to (13) the point $\tilde{k}_q$ belongs to the interval of definition of the function $C(k_q)$. Now we calculate the value of income for $k_q = \tilde{k}_q$. It is equal to $\tilde{C} = C(\tilde{k}_q)$, where

$$\tilde{C} = qp\left(\frac{eA(1-j)K_p + (1-i)}{2Ae(1-j)}\right)\left(1 - i + \frac{eA(1-j)K_p - (1-i)}{2}\right).$$

As a result of simplifications, we obtain:

$$\tilde{C} = \frac{qp}{4Ae(1-j)}\left(eA(1-j)K_p + 1 - i\right)^2, \ 0 < j < i. \tag{15}$$

Expression (15) means the maximum income of an entrepreneur who has accepted the government's proposal to expand production. It is achieved when the coefficient of sales increase k_q is equal to $\tilde{k}_q$, which is defined by the formula (14).

Dependence of the maximum income on the preferential tax rate

It should be noted that the state can influence the value of the maximum income of the entrepreneur (15) by choosing a preferential tax rate within the range of variation j that satisfies the system of inequalities (11). This influence on the value (15) can be either upward or downward. In this case, the area of influence of the tax rate j is subject to the system of inequalities (11), where $k_q = \tilde{k}_q$. Namely, we have a system of constraints for the parameter j in the form:

$$\begin{cases} j < 1 - \dfrac{1-i}{A\left(eK_p - \tilde{k}_q\right)}, \\ 0 < j < i. \end{cases} \tag{16}$$

We transform the first of the inequalities (16). We have:

$$j < 1 - \frac{1-i}{A\left(eK_p - \tilde{k}_q\right)} \ \Leftrightarrow \ (1-j)A\left(eK_p - \tilde{k}_q\right) > 1 - i \ \Rightarrow 1 - j > \frac{1-i}{eAK_p}.$$

As a result, we obtain the condition $j < 1 - \dfrac{1-i}{eAK_p}$. Thus, the required domain of definition of the variable j isn't larger than the domain defined by the following constraints:

$$0 < j < i, \ j < 1 - \frac{1-i}{eAK_p}.$$

This system of inequalities makes sense only if the right side of the second inequality is positive. Namely, the following relations must be fulfilled:

$$1 - \frac{1-i}{eAK_p} > 0 \ \Leftrightarrow \ eAK_p > 1 - i \ \Leftrightarrow \ e > \frac{1-i}{AK_p}.$$

Consequently, we can consider the function $\varphi(j)$ defined by (15) in the form:

$$\varphi(j) = \frac{qp}{4Ae(1-j)}\left(eA(1-j)K_p + 1 - i\right)^2, \tag{17}$$

which is given on the set of values of the variable j:

$$\begin{cases} 0 < j < i, \\ j < 1 - \dfrac{1-i}{eAK_p} \end{cases} \text{for } e > \dfrac{1-i}{AK_p}. \tag{18}$$

We transform expression (17) as follows:

$$\varphi(j) = \frac{qp}{4Ae}\left(\left(eAK_p\right)^2 (1-j) + 2eAK_p(1-i) + \frac{(1-i)^2}{1-j}\right).$$

From this we easily find the derivatives of the function (17). They are equal:

$$\frac{d\varphi(j)}{dj} = \frac{qp}{4Ae}\left(-\left(eAK_p\right)^2 + \frac{(1-i)^2}{(1-j)^2}\right); \quad \frac{d^2\varphi(j)}{dj^2} = \frac{qp}{4Ae}\frac{2(1-i)^2}{(1-j)^3}.$$

The first derivative is equal to zero when the following relations are satisfied:

$$\frac{1-i}{eAK_p} - (1-j) = 0 \iff j = \hat{j} = \frac{eAK_p - (1-i)}{eAK_p}.$$

Taking into account the system of inequalities (18), we obtain that $\hat{j} > i$, $\left(1 - \dfrac{1-i}{eAK_p} > i \iff 1 - i > \dfrac{1-i}{eAK_p}\right)$.

This means that the point $j = \hat{j}$ is located to the right of the definition interval of the function $\varphi(j)$. Therefore, the derivative is negative, and hence the function decreases for all $0 < j < \hat{j}$. In addition, the following limit equations hold:

$$\lim_{j \to +0} \varphi(j) = \frac{qp}{4Ae}\left(eAK_p + 1 - i\right)^2, \quad \lim_{j \to i-0} \varphi(j) = \frac{qp(1-i)}{4Ae}\left(eAK_p + 1\right)^2.$$

Moreover, the second derivative is positive, and hence the function $\varphi(j)$ is convex.

The graph of the function is shown in fig. 1, where $\tilde{\varphi} = qp\dfrac{\left(eAK_p + 1 - i\right)^2}{4Ae}$ and in accordance with conditions (18) $\overline{j} = \min\left\{1 - \dfrac{1-i}{eAK_p},\ i\right\}$.
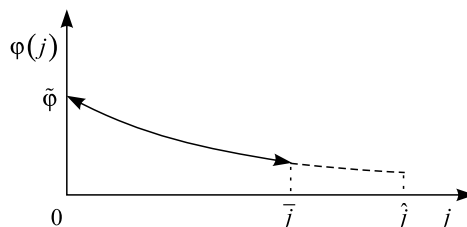

Fig. 1. The graph of the maximum income function $\varphi(j) = \tilde{C}$

The figure confirms the increase in the maximum income with a lower tax rate j.

Conditions (17), (18) mean that the use of a preferential tax rate j by the government creates conditions for a real increase in the entrepreneur's income, on the one hand, and on the other hand, the government has the ability to regulate the largest of such allowable incomes towards reducing it.

Notes.

1. *If the tax rate j is absent (the tax on the increased part of production is not withdrawn), then the entrepreneur's income has the best value equal to*

$$\tilde{\varphi} = \frac{qp}{4Ae}\left(eAK_p + 1 - i\right)^2.$$

2. *If the domain of definition of the function* $\varphi(j)$ *is an empty set, i. e. there is a condition for the price elasticity of demand in the form*

$$0 < e \le \frac{1-i}{AK_p},$$

then the government can't influence the maximum of the entrepreneur's income This fact corresponds to markets with relatively low price elasticity of demand.

Dependence of the maximum income on the rate of price growth

According to the maximum income formula (15), we consider the function:

$$\alpha(x) = \frac{qp}{4Ae(1-j)}\left(eA(1-j)x+1-i\right)^2, \ 0 < j < i,$$

where the notation $x = K_p$ for the rate of price growth is used. The domain of definition of this function is given by the condition (13). From this we obtain the restrictions on the variable x in the form of the following relations:

$$0 < k_q < ex - \frac{1-i}{A(1-j)} \Leftrightarrow x > \frac{A(1-j)k_q+1-i}{eA(1-j)}.$$

The function $\alpha(x)$ is quadratic, its graph is the part of the parabola with the branches directed upwards. The vertex of the parabola corresponds to the value $x = -\dfrac{1-i}{eA(1-j)} < 0$.

We calculate the value of the function at the extreme left point $x = \bar{x} = \dfrac{A(1-j)k_q+1-i}{eA(1-j)}$ of the interval of its definition. We obtain:

$$eA(1-j)\bar{x}+1-i = eA(1-j)\frac{A(1-j)k_q+1-i}{eA(1-j)}+1-i = A(1-j)k_q+2(1-i).$$

Therefore, at this point we have

$$\alpha(\bar{x}) = \frac{qp}{4Ae(1-j)}\left(A(1-j)k_q+2(1-i)\right)^2.$$

The graph of the maximum income function $\alpha(x)$ is shown in fig. 2.
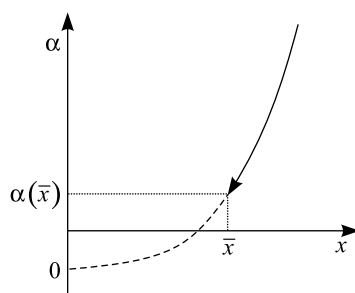

Fig. 2. The graph of the maximum income function $\alpha(x) = \tilde{C}, \ x = K_p$

The figure shows that an increase in the rate of price growth, of course, leads to an increase in the maximum income of the entrepreneur. This means an increase in the level of inflation, which leads in turn to the growth of only the nominal value of income, i. e. to the depreciation of money.

Dependence of the maximum income on the price elasticity of demand

The formula of the maximum income $\tilde{C}$ can be considered as a function of the variable e. Therefore, it is interesting to find out the dependence of the function $\tilde{C}$ on the type of market taking into account the price elasticity of demand. It should be reminded that the parameter e is subject to the condition (8), more precisely

to the inequality $eK_p > k_q$, where $k_q = \tilde{k}_q = \dfrac{1}{2} eK_p - \dfrac{1-i}{2A(1-j)}$. Hence, the following inequality must be fulfilled with respect to the variable e:

$$eK_p > \frac{1}{2} eK_p - \frac{1-i}{2A(1-j)}.$$

This inequality holds for all $e > 0$. Thus, using the formula (15), we can consider the function

$$\sigma(e) = \frac{qp}{4eA(1-j)}\left(eA(1-j)K_p + 1 - i\right)^2, \ e > 0, \tag{19}$$

for $0 < j < i$. We transform the expression for $\sigma(e)$. As a result, we obtain the function

$$\sigma(e) = \frac{qp}{4A(1-j)}\left(e(A(1-j)K_p)^2 + 2A(1-j)(1-i)K_p + \frac{(1-i)^2}{e}\right), \ e > 0.$$

Hence we compute the first and second derivatives of the function (19). They have the form:

$$\frac{d\sigma(e)}{de} = \frac{qp}{4A(1-j)}\left((A(1-j)K_p)^2 - \frac{(1-i)^2}{e^2}\right), \ \frac{d^2\sigma(e)}{de^2} = \frac{qp(1-i)^2}{2A(1-j)e^3}, \ e > 0.$$

The first derivative is equal to zero in the domain of definition $e > 0$ at the point

$$e = \hat{e} = \frac{1-i}{A(1-j)K_p}.$$

It is clear that the derivative is negative for $0 < e < \hat{e}$ and it is positive for $e > \hat{e}$. This means that the function $\sigma(e)$ decreases in the first case and increases in the second case. In addition, the second derivative is positive, i. e. the graph of the function is convex. We calculate the following limit values:

$$\lim_{e \to +0} \sigma(e) = +\infty, \ \lim_{e \to +\infty} \sigma(e) = +\infty.$$

Research shows, that at the point $e = \hat{e} = \dfrac{1-i}{A(1-j)K_p}$, where the sign of the derivative changes, the function $\sigma(e)$ reaches its minimum. Let us calculate this minimum value:

$$\sigma(\hat{e}) = \frac{qp}{4A(1-j)}\left(\hat{e}\left(A(1-j)K_p\right)^2 + 2A(1-j)(1-i)K_p + \frac{(1-i)^2}{\hat{e}}\right) = qp(1-i)K_p.$$

The graph of the function $\sigma(e)$ is shown in fig. 3, where $\hat{e} = \dfrac{1-i}{A(1-j)K_p}$.
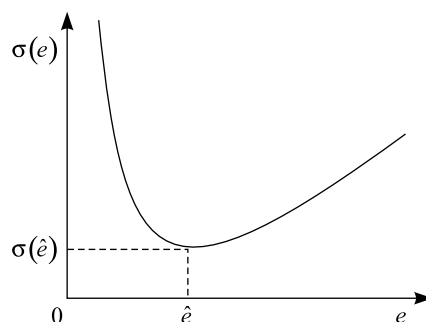

Fig. 3. The graph of the maximum income function $\sigma(e) = \tilde{C}$

The graph shows that, depending on the absolute value of the coefficient of price elasticity of demand e, the maximum income of the entrepreneur decreases with changing e in the interval $0 < e < \dfrac{1-i}{A(1-j)K_p}$ and then increases for $e > \dfrac{1-i}{A(1-j)K_p}$.

Dependence of the maximum income on the coefficient of costs

Consider the maximum income formula $\tilde{C}$ (15) as a function $\beta(a)$ of the variable a:

$$\beta(a) = \frac{qp}{4e(1-a)(1-j)}\big(e(1-a)(1-j)K_p + 1 - i\big)^2, \ \ 0 < a < 1. \tag{20}$$

In this case, we have the following limits:

$$\lim_{a \to +0} \beta(a) = \frac{qp}{4e(1-j)}\big(e(1-j)K_p + 1 - i\big)^2, \ \ \lim_{a \to 1-0} \beta(a) = +\infty.$$

Let us determine the value a at which the function $\beta(a)$ represented by the formula (20) is equal to zero. We have $e(1-a)(1-j)K_p + 1 - i = 0,$ where $a = a^*$. The quantity a^* is equal to:

$$a^* = 1 + \frac{1-i}{e(1-j)K_p}.$$

The maximum income function $\beta(a)$ has an economic sense only on the interval $0 < a < 1$ and the value a^* is outside the domain of definition of the function, because $a^* > 1$.

To determine the extreme values of the function $\beta(a)$, which is represented by the formula (20), we transform it as follows:

$$\beta(a) = \frac{qp}{4}\left(e(1-a)(1-j)K_p^2 + \frac{(1-i)^2}{e(1-a)(1-j)} + 2(1-i)K_p\right). \tag{21}$$

Next, let's find the derivative of the function (21), for this we equate it to zero and write down the roots:

$$\frac{d\beta(a)}{da} = \frac{qp}{4}\left(-e(1-j)K_p^2 + \frac{(1-i)^2}{e(1-a)^2(1-j)}\right) = 0;$$

$$a_1 = 1 - \frac{1-i}{e(1-j)K_p}; \ \ a_2 = 1 + \frac{1-i}{e(1-j)K_p}.$$

One of the found roots does not belong to the domain of definition of the function, namely $a = a_2$, moreover $a_2 = a^*$. It is clear that the derivative is negative on the interval $0 < a < a_1$ and it is positive for $a_1 < a < 1$. Therefore, the function has a minimum at the point $a = a_1$. We calculate the value of the function $\beta(a)$ at the points a_1 and $a = 0$. We have:

$$\beta(a_1) = qp(1-i)K_p > 0.$$

$$\beta(0) = \frac{qp}{4e(1-j)}\big(e(1-j)K_p + 1 - i\big)^2, \ \ \beta(0) > 0.$$

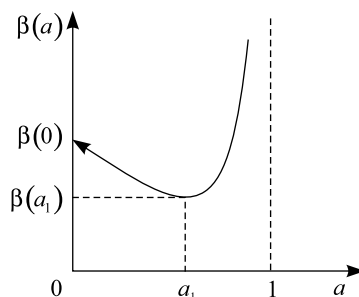The graph of the maximum income function of the enterprise $\beta(a)$ is shown in fig. 4:

Fig. 4. The graph of the maximum income function $\beta(a) = \tilde{C}$

Based on the obtained graph, we can conclude that the value of the maximum income decreases as the indicator a grows to the value a_1. Further, with an increase in the coefficient of costs in the production of an additional unit of products, there is a growth in the possibilities for increasing the maximum income of the entrepreneur.

Conclusion

This paper is devoted to the study of incentives used by the state to influence the increase in the output of the entrepreneur. As such incentives, the state uses preferential tax rates.

It is revealed that the maximum income of an entrepreneur who accepted the government's proposal to expand production is represented by the expression (15). It is achieved when the coefficient of sales increase k_q is equal to $\tilde{k}_q$, which determined by the formula (14).

The maximum value of the entrepreneur's income is influenced by such factors as the preferential tax rate, the rate of price growth, price elasticity of demand and the coefficient of costs. As a result, the following conditions are obtained for the maximum benefit of the enterprise from the increase in the output:
- the entrepreneur's income increases when the preferential tax rate tends to zero (fig. 1);
- the increase in the rate of price growth leads to an increase in the maximum amount of the entrepreneur's income (fig. 2);
- the entrepreneur's income reaches its maximum value in the markets of goods with price elasticity close to zero or in the markets of highly elastic goods (fig. 3);
- the higher the coefficient of costs in the production of an additional unit of products, the greater the benefit which the enterprise receives from increasing output (fig. 4).

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