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О КОНСТАНТЕ ЛЕБЕГА ИНТЕРПОЛЯЦИОННОГО РАЦИОНАЛЬНОГО ПРОЦЕССА С УЗЛАМИ ЧЕБЫШЕВА – МАРКОВА

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Рассматривается оценка константы Лебега интерполяционного рационального процесса Лагранжа на отрезке $[-1, 1]$ с узлами в нулях косинус-дробей Чебышева – Маркова. Показано, что в случае двух действительных геометрически различных полюсов аппроксимирующих функций нормы фундаментальных многочленов Лагранжа ограниченны. На основании этого результата доказано, что в рассматриваемом случае оценка сверху константы Лебега не зависит от расположения полюсов и последовательность констант Лебега растет с логарифмической скоростью. В предыдущих работах оценки констант Лебега были получены только для конкретных наборов полюсов или зависели от расположения полюсов.

Ключевые слова: рациональные приближения; интерполирование; дробь Чебышева – Маркова; константа Лебега.

ON A LEBESGUE CONSTANT OF INTERPOLATION RATIONAL PROCESS AT THE CHEBYSHEV – MARKOV NODES

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In the present paper estimate of a Lebesgue constant of the interpolation rational Lagrange process on the segment $[-1, 1]$ at the Chebyshev – Markov cosine fractions nodes is considered. It is shown that in the case of two real geometrically distinct poles of approximating functions, the norms of the Lagrange fundamental polynomials are bounded. Based on this result, it is proved that in the case under consideration the upper estimate of the Lebesgue constant does not depend on the arrangement of the poles and the sequence of the Lebesgue constant grows with logarithmic rate. Note, that in previous works the estimates of Lebesgue constants were obtained only for particular choices of poles or depended on the arrangement of poles.

Key words: rational approximation; interpolation; Chebyshev – Markov fraction; Lebesgue constant.

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Introduction

Behaviour of the Lebesgue constant of interpolating polynomial processes was always of great interest of the researchers (see, for example, [1]). In this direction very deep and profound results were obtained [2].

In the case of interpolation by rational functions, this problem is less studied. Beginning of the investigation of the behavior of the Lebesgue constants of interpolation rational Lagrange processes at the Chebyshev – Markov nodes on a segment was made by V. N. Rusak [3], continued by A. P. Starovoitov [4]. However, in these papers estimates of the Lebesgue constants were obtained under certain conditions on the poles of interpolation rational functions. The main result of [5] is of a different nature. Let us quote it.

Let $\{a_k\}_{k=0}^{n}$ be a sequence of real numbers such that $a_0 = 0,\ a_k \in (-1, 1),\ k = 1, 2, \ldots, n.$

Chebyshev – Markov rational fraction can be defined as follows

$$M_n(x) = \cos\mu_n(x),$$

where

$$\mu_n(x) = \sum_{k=0}^{n} \arccos\frac{x + a_k}{1 + a_k x},$$

and

$$\mu_n'(x) = -\frac{\lambda_n(x)}{\sqrt{1 - x^2}},\ \lambda_n(x) = \sum_{k=0}^{n}\frac{\sqrt{1 - a_k^2}}{1 + a_k x},\ x \in (-1, 1).$$

The function $M_n(x)$ has $n + 1$ zeroes x_k, $k = 0, 1, \ldots, n$, on the interval $(-1, 1)$. Then for any function f defined on $[-1, 1]$ we construct the interpolating rational Lagrange function

$$L_n(x; f) = \sum_{k=0}^{n} f(x_k)\, l_k(x), \tag{1}$$

where

$$l_k(x) = \frac{M_n(x)}{(x - x_k)M_n'(x_k)},\ k = 0, 1, \ldots, n.$$

Theorem 1. *For the Lebesgue function*

$$L_n(x) = \sum_{k=0}^{n} |l_k(x)|$$

of the interpolating Lagrange process (1) the following estimate holds

$$L_n(x) < 1 + Cd_n \ln\sum_{k=0}^{n}\left(\sqrt{\frac{1 - a_k}{1 + a_k}} + \sqrt{\frac{1 + a_k}{1 - a_k}}\right),\ n = 1, 2, \ldots, \tag{2}$$

where C is a positive constant,

$$d_n = \max_{k = 1, 2, \ldots, n}\left\{\frac{\lambda_n(x_{k-1})}{\lambda_n(x_k)}, \frac{\lambda_n(x_k)}{\lambda_n(x_{k-1})}\right\}.$$

Note, that estimate (2) was obtained for any real poles, without any restrictions on their location.

Let us pay some attention to the right-hand side of the estimate (2). Like in previous studies it is a function of poles. However in [6] it was proved that in one particular case $n = 2$ there is no such dependence. In the present work we continue such study.

Basic definitions and auxiliary results

Let $a \in [0, 1)$. Chebyshev – Markov rational fraction with two geometrically distinct real poles can be written as follows:

$$M_{2n}(x) = \cos\mu_{2n}(x),\ x \in [-1, 1], \tag{3}$$

$$\mu_{2n}(x) = n\left(\arccos\frac{x + a}{1 + ax} + \arccos\frac{x - a}{1 - ax}\right).$$

Remark 1. If

$$x = \frac{\cos\theta}{\sqrt{1 - a^2 \sin^2\theta}}, \ \ \theta \in [0, \pi],$$

then the rational fraction (1) can be written in the following form

$$M_{2n}\left(\frac{\cos\theta}{\sqrt{1 - a^2 \sin^2\theta}}\right) = \cos 2n\theta, \ \ \theta \in [0, \pi].$$

Using the definition, we obtain directly the following lemma.

Lemma 1. *The function* $M_{2n}(x)$ *has* $2n$ *simple zeroes,* x_k, $k = 1, 2, \ldots, 2n$ *on the interval* $(-1, 1)$, *that are symmetric with respect to the imaginary axis:*

$$-1 < x_{2n} < x_{2n-1} < \ldots < x_{n+1} < 0 < x_n < x_{n-1} < \ldots < x_1 < 1;$$

$$x_k = \frac{\cos\theta_k}{\sqrt{1 - a^2 \sin^2\theta_k}}, \ \ \theta_k = \frac{2k-1}{4n}\pi, \ \ k = 1, 2, \ldots, 2n;$$

$$x_{n+k} = -x_{n-k+1}, \ \ k = 1, 2, \ldots, n.$$

For the further research we need the following property of the zeroes of Chebyshev – Markov rational fraction (3).

Lemma 2. *The following inequalities hold:*

$$\frac{1 - a^2 \sin^2\theta_{k-1}}{1 - a^2 \sin^2\theta_k} < 9, \ \ a \in [0, 1), \ \ \theta_k = \frac{2k-1}{4n}\pi, \ \ k = 2, 3, \ldots, n+1.$$

P r o o f. Let us consider the functions

$$\varphi_k(u) = \frac{1 - u \sin^2\theta_{k-1}}{1 - u \sin^2\theta_k}, \ \ u \in [0, 1), \ \ k = 2, 3, \ldots, n.$$

Since

$$\varphi_k'(u) = \frac{\sin^2\theta_k - \sin^2\theta_{k-1}}{\left(1 - u \sin^2\theta_k\right)^2} > 0,$$

then

$$\frac{1 - a^2 \sin^2\theta_{k-1}}{1 - a^2 \sin^2\theta_k} < \frac{1 - \sin^2\theta_{k-1}}{1 - \sin^2\theta_k} = \left(\frac{\cos\theta_{k-1}}{\cos\theta_k}\right)^2.$$

Now,

$$\left(\frac{\cos\left(\theta - \frac{\pi}{2n}\right)}{\cos\theta}\right)' = \frac{\sin\left(\frac{\pi}{2n}\right)}{\cos^2\theta} > 0, \ \ \theta \in \left[0, \frac{\pi}{2}\right),$$

and

$$\left(\frac{\cos\theta_{k-1}}{\cos\theta_k}\right)^2 < \left(\frac{\cos\theta_{n-1}}{\cos\theta_n}\right)^2 = \left(\frac{\sin\left(\frac{3\pi}{4n}\right)}{\sin\left(\frac{\pi}{4n}\right)}\right)^2 < 9.$$

We have $\theta_{n+1} + \theta_n = \pi$ for $k = n + 1$. It means

$$\frac{1-a^2\sin^2\theta_n}{1-a^2\sin^2\theta_{n+1}}=1.$$

Lemma 2 is proved.

Now for any function $f(x)$, defined on the segment $[-1, 1]$, we construct the interpolating rational Lagrange function

$$L_{2n}(x; f)=\sum_{k=1}^{2n} f(x_k)\,l_k(x), \tag{4}$$

where

$$l_k(x)=\frac{M_{2n}(x)}{(x-x_k)M'_{2n}(x_k)},\ \ k=1, 2, \ldots, 2n. \tag{5}$$

It turns out that in this case the uniform norms of the Lagrange's fundamental polynomials are bounded.

Lemma 3. *The following inequalities hold:*

$$\max_{x\in[-1,1]}|l_k(x)|<6,\ \ k=1, 2, \ldots, n+1.$$

P r o o f. Since

$$M'_{2n}(x)=\sin\mu_{2n}(x)\frac{2n\sqrt{1-a^2}}{(1-a^2x^2)\sqrt{1-x^2}},$$

then from (5) it follows that

$$|l_k(x)|=\frac{|M_{2n}(x)|(1-a^2x_k^2)\sqrt{1-x_k^2}}{2n\sqrt{1-a^2}\,|x-x_k|}. \tag{6}$$

Using lemma 2, it is not difficult to check, that

$$1-a^2x_k^2=1-\frac{a^2\cos^2\theta_k}{1-a^2\sin^2\theta_k}=\frac{1-a^2}{1-a^2\sin^2\theta_k}, \tag{7}$$

$$\sqrt{1-x_k^2}=\sqrt{1-\frac{\cos^2\theta_k}{1-a^2\sin^2\theta_k}}=\frac{\sin\theta_k\sqrt{1-a^2}}{1-a^2\sin^2\theta_k}. \tag{8}$$

Besides, assuming

$$x=\frac{\cos\theta}{\sqrt{1-a^2\sin^2\theta}},\ \ \theta\in[0, \pi],$$

we obtain

$$x-x_k=\frac{\cos\theta}{\sqrt{1-a^2\sin^2\theta}}-\frac{\cos\theta_k}{\sqrt{1-a^2\sin^2\theta_k}}=\frac{\cos\theta\sqrt{1-a^2\sin^2\theta_k}-\cos\theta_k\sqrt{1-a^2\sin^2\theta}}{\sqrt{1-a^2\sin^2\theta}\sqrt{1-a^2\sin^2\theta_k}}=$$

$$=\frac{\cos^2\theta(1-a^2\sin^2\theta_k)-\cos^2\theta_k(1-a^2\sin^2\theta)}{\sqrt{1-a^2\sin^2\theta}\sqrt{1-a^2\sin^2\theta_k}\left(\cos\theta\sqrt{1-a^2\sin^2\theta_k}+\cos\theta_k\sqrt{1-a^2\sin^2\theta}\right)}.$$

Then

$$\cos^2\theta(1-a^2\sin^2\theta_k)-\cos^2\theta_k(1-a^2\sin^2\theta)=\cos^2\theta-a^2\cos^2\theta(1-a^2\cos^2\theta_k)-$$

$$-\cos^2\theta_k+a^2\cos^2\theta_k(1-a^2\cos^2\theta)=(1-a^2)(\cos^2\theta-\cos^2\theta_k).$$

Therefore,

$$x - x_k = \frac{\left(1 - a^2\right)\left(\cos^2\theta - \cos^2\theta_k\right)}{\sqrt{1 - a^2\sin^2\theta}\sqrt{1 - a^2\sin^2\theta_k}\left(\cos\theta\sqrt{1 - a^2\sin^2\theta_k} + \cos\theta_k\sqrt{1 - a^2\sin^2\theta}\right)}. \quad (9)$$

It is clear, that

$$\left|l_k(x)\right| = \left|l_k\left(\frac{\cos\theta}{\sqrt{1 - a^2\sin^2\theta}}\right)\right|.$$

Then, plugging (7)–(9) into (6), using remark 1, we obtain

$$\left|l_k(x)\right| = \frac{\left|\cos 2n\theta\right| \sin\theta_k\sqrt{1 - a^2\sin^2\theta}\left|\cos\theta\sqrt{1 - a^2\sin^2\theta_k} + \cos\theta_k\sqrt{1 - a^2\sin^2\theta}\right|}{2n\left(1 - a^2\sin^2\theta_k\right)\sqrt{1 - a^2\sin^2\theta}\left|\cos^2\theta - \cos^2\theta_k\right|}. \quad (10)$$

Now, let $x \in [0, 1]$ and j, $2 \le j \le n$, be such an index that $x \in \left(x_j, x_{j-1}\right)$. Let us pay attention to the equality (10). If $k \le j - 1$, then

$$\left|\cos\theta\sqrt{1 - a^2\sin^2\theta_k} + \cos\theta_k\sqrt{1 - a^2\sin^2\theta}\right| < \sqrt{1 - a^2\sin^2\theta_k}\left(\cos\theta + \cos\theta_k\right).$$

Therefore,

$$\left|l_k(x)\right| < \frac{\left|\cos 2n\theta\right| \sin\theta_k\sqrt{1 - a^2\sin^2\theta}}{2n\sqrt{1 - a^2\sin^2\theta_k}\left(\cos\theta_k - \cos\theta\right)} < \frac{\left|\cos 2n\theta\right|\sin\theta_k}{2n\left(\cos\theta_k - \cos\theta\right)}\sqrt{\frac{1 - a^2\sin^2\theta_j}{1 - a^2\sin^2\theta_k}}.$$

The estimate for the first factor in the right-hand side of the last inequality can be found, for example in [7, p. 528]. For the second factor we use lemma 3. Then

$$\frac{1 - a^2\sin^2\theta_j}{1 - a^2\sin^2\theta_k} \le \frac{1 - a^2\sin^2\theta_j}{1 - a^2\sin^2\theta_{j-1}} < 9.$$

Finally,

$$\left|l_k(x)\right| < 9, \ k \le j - 1.$$

Then let $j \le k \le n$. In this case the factor in the numerator of (10) can be estimated as follows

$$\left|\cos\theta\sqrt{1 - a^2\sin^2\theta_k} + \cos\theta_k\sqrt{1 - a^2\sin^2\theta}\right| < \sqrt{1 - a^2\sin^2\theta}\left(\cos\theta + \cos\theta_k\right).$$

Thus,

$$\left|l_k(x)\right| < \frac{\left|\cos 2n\theta\right|\sin\theta_k}{2n\left(\cos\theta_k - \cos\theta\right)}\frac{1 - a^2\sin^2\theta}{1 - a^2\sin^2\theta_k}.$$

It is no difficult to check that the second factor in the right-hand side of the last inequality is not greater than 1 and therefore we obtain

$$\left|l_k(x)\right| < 2, \ j \le k \le n.$$

So, we proved lemma when $x \in \left(x_n, x_1\right)$. If $x \in \left[0, x_n\right] \cup \left[x_1, 1\right]$ then in the scheme of the proof of the lemma, there will be obvious changes. Besides, it is clear that

$$\max_{x \in [0,1]}\left|l_k(x)\right| > \max_{x \in [-1,0]}\left|l_k(x)\right|.$$

This concludes the proof.

The main result

According to the definition Lebesgue function of an interpolating process is as follows

$$\Lambda_{2n}(x) = \sum_{k=1}^{2n}\left|l_k(x)\right|,$$

and Lebesgue constant is defined as

$$\Lambda_{2n} = \max_{x \in [-1, 1]} \Lambda_{2n}(x), \ \ k = 1, 2, \ldots.$$

Theorem 2. *For the Lebesgue constant of the interpolating rational Lagrange process (4) the following estimate holds*

$$\Lambda_{2n} \le C_1(1 + \ln n), \ \ n = 1, 2, \ldots,$$

where C_1 is a positive constant.

Proof. Here we use modification of the method, applied by S. N. Bernstein to estimate the Lebesgue constant in the polynomial case (see, for example, [7, p. 539]). We are going to check the inequality $\Lambda_{2n}(x) \le C_1(1 + \ln n)$ for arbitrary $x \in [-1, 1]$. First, it is clear, that if $x = x_k$, $k = 1, 2, \ldots, 2n$, then $\Lambda_{2n}(x_k) = 1$.

Now, let $x \in [0, 1]$. Since the Chebyshev – Markov rational fraction (3) is even,

$$\Lambda_{2n}(x) \le 2\sum_{k=1}^{n} |l_k(x)|.$$

Let j, $j = 1, 2, \ldots, n$, be a number such that $x_j < x < x_{j-1}$ (assume that $x_0 = 1$). We represent the sum in the right-hand side of the last equality in the form

$$\Lambda_{2n}(x) \le 2\left(S_1(x) + \sum_{k=j-2}^{j+1} |l_k(x)| + S_2(x) \right),$$

where

$$S_1(x) = \sum_{k=1}^{j-3} |l_k(x)|, \ \ S_2(x) = \sum_{k=j+2}^{n} |l_k(x)|.$$

Note that the $S_1(x)$ vanishes for $j = 1, 2, 3$, and the sum $S_2(x)$ vanishes for $j = n - 1, n$. Applying lemma 3 we obtain

$$\Lambda_{2n}(x) \le 2\big(S_1(x) + S_2(x) + 24\big). \tag{11}$$

Let us consider the sum $S_1(x)$. Using definitions of Chebyshev – Markov rational fractions (3) and Lagrange's fundamental polynomials (5), it is easy to see that for $k = 1, 2, \ldots, j - 3$

$$|l_k(x)| = \frac{\left(1 - a^2 x_k^2\right)\sqrt{1 - x_k^2}}{2n\sqrt{1 - a^2}\,(x_k - x)}.$$

Assuming $x = \cos\theta$ and applying lemma 2, we obtain

$$|l_k(x)| \le \frac{\sin\theta_k}{2n\left(1 - a^2\sin^2\theta_k\right)} \frac{\sqrt{1 - a^2\sin^2\theta}\left(\cos\theta\sqrt{1 - a^2\sin^2\theta_k} + \cos\theta_k\sqrt{1 - a^2\sin^2\theta}\right)}{\cos^2\theta_k - \cos^2\theta}.$$

Given the location of the points θ_k, $k = 1, 2, \ldots, j - 3$, the last estimate can be transformed as follows:

$$|l_k(x)| \le \frac{\sin\theta_k}{2n\left(1 - a^2\sin^2\theta_k\right)} \frac{\sqrt{1 - a^2\sin^2\theta}\sqrt{1 - a^2\sin^2\theta_k}\,\left(\cos\theta + \cos\theta_k\right)}{\cos^2\theta_k - \cos^2\theta} \le \frac{\sin\theta_k}{2n(\cos\theta_k - \cos\theta)}.$$

Therefore,

$$S_1(x) \le \frac{1}{2n}\sum_{k=1}^{j-3} \frac{\sin\theta_k}{\cos\theta_k - \cos\theta}.$$

Then, since the function $\dfrac{\sin x}{\cos x - \cos\theta}$ increases for $x \in [0, \theta)$,

$$\frac{\sin\theta_k}{\cos\theta_k - \cos\theta} \le \frac{\sin x}{\cos x - \cos\theta}, \; x \in \left[\theta_k, \theta_{k+1}\right].$$

Integrating the last inequality with respect to x from θ_k to θ_{k+1}, we get

$$\frac{\pi}{2n}\frac{\sin\theta_k}{\cos\theta_k - \cos\theta} \le \int\limits_{\theta_k}^{\theta_{k+1}} \frac{\sin x\,dx}{\cos x - \cos\theta}.$$

Then

$$S_1(x) \le \frac{1}{\pi}\int\limits_{\theta_1}^{\theta_{j-2}} \frac{\sin x\,dx}{\cos x - \cos\theta} < \frac{1}{\pi}\int\limits_{0}^{\theta_{j-2}} \frac{\sin x\,dx}{\cos x - \cos\theta} = \frac{1}{\pi}\ln\frac{1-\cos\theta}{\cos\theta_{j-2} - \cos\theta}.$$

Further, we note that

$$1 - \cos\theta \le 2, \; \cos\theta_{j-2} - \cos\theta > \cos\theta_{j-2} - \cos\theta_{j-1}.$$

From here,

$$S_1(x) \le -\frac{1}{\pi}\left(\ln\sin\frac{\theta_{j-2}+\theta_{j-1}}{2} + \ln\sin\frac{\pi}{4n}\right).$$

It remains to note that, $\theta_{j-1} > \theta_{j-2} > \theta_1$ and therefore

$$\ln\sin\frac{\theta_{j-2}+\theta_{j-1}}{2} > \ln\sin\theta_1 = \ln\sin\frac{\pi}{4n}.$$

Finally for the sum $S_1(x)$ we obtain,

$$S_1(x) \le -\frac{2}{\pi}\ln\sin\frac{\pi}{4n} \le -\frac{2}{\pi}\ln\frac{1}{2n} = \frac{2}{\pi}\ln n + \frac{2\ln 2}{\pi}.$$

A similar estimate also holds for the sum $S_2(x)$. Plugging the obtained results in the inequality (11), we conclude that

$$\Lambda_{2n}(x) \le \frac{8}{\pi}\ln n + \frac{8\ln 2}{\pi} + 48.$$

The case $x \in [-1, 0]$ can be considered in the same way. Theorem 2 is proved.

Numerical examples

The result of theorem 2 shows that in the case of two geometrically distinct real poles the behavior of the Lebesgue constant does not depend on the choice of the parameters $\{a_k\}_{k=0}^{n}$. In this section we show results of some numerical experiments for calculation of the Lebesgue function and the Lebesgue constant in the considered case of the choice of the parameters as well as some other cases. Numerical experiment was carried out using programming language *Python*, version 3.6, with the help of libraries *NumPy*, *SciPy* and *Matplotlib*.

Two real poles

Let n be even number, $a_0 = 0$, $a_k = \frac{1}{2}$, $k = 1, 2, \ldots, \frac{n}{2}$, $a_k = \frac{-1}{2}$, $k = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$. Then the graph of the function $\frac{\Lambda_{2n}(x)}{1+\ln n}$ for $n = 30$ is presented on the fig. 1 (horizontal axis – values of x, vertical axis – values of the function $\frac{\Lambda_{2n}(x)}{1+\ln n}$).

Behavior of the Lebesgue constant in this case is described by the fig. 2 (horizontal axis – values of n, vertical axis – values of the ratio $\frac{\Lambda_{2n}}{1+\ln n}$).

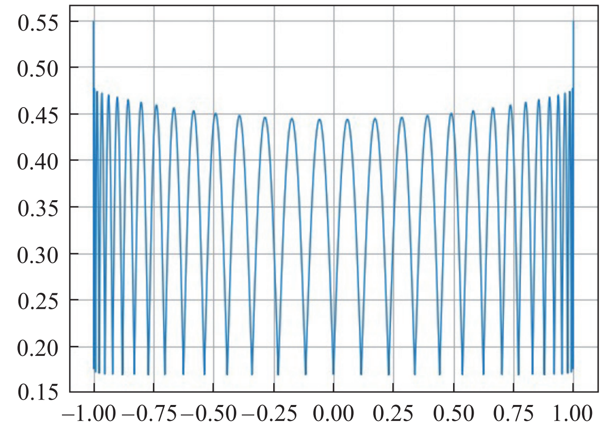

Fig. 1. Graph of $\frac{\Lambda_{2n}(x)}{1+\ln n}$ for the case of two real poles and $n = 30$

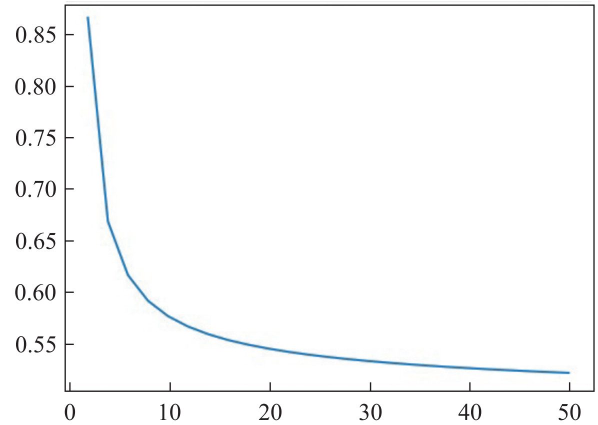

Fig. 2. Lebesgue constant for the case of two real poles

The results of the experiment support the conclusions of theorem 2 and even allow us to make an assumption about the value of the constant in the statement of this theorem.

Also in the experiment we considered many other cases. Here the results of some of them.

Real poles

Let n be even number, $a_0 = 0$, $a_k = e^{-1/n}$, $k = 1, 2, \ldots, \frac{n}{2}$, $a_k = -e^{-1/n}$, $k = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$. Then the graph of the function $\frac{\Lambda_{2n}(x)}{1+\ln n}$ for $n = 30$ is presented on the fig. 3 (axes mean the same as in the subsection «Two real poles»).

Behavior of the Lebesgue constant in this case is described by the fig. 4.

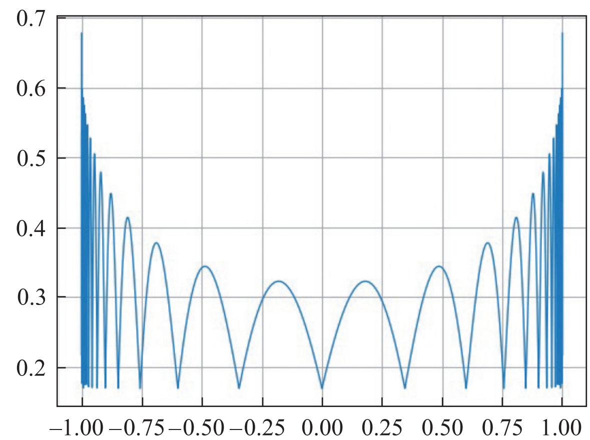

Fig. 3. Graph of $\frac{\Lambda_{2n}(x)}{1+\ln n}$ for the case of real poles and $n = 30$

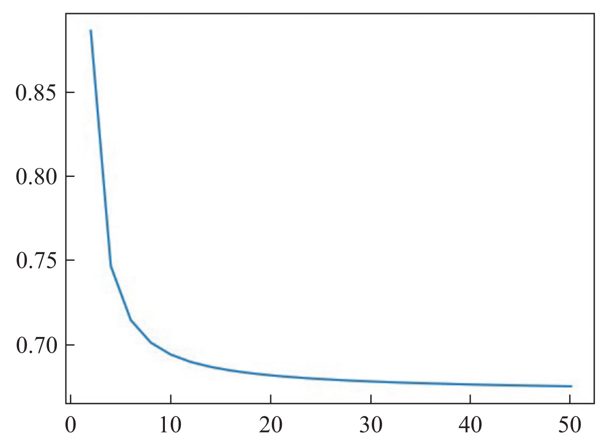

Fig. 4. Lebesgue constant for the case of real poles

Complex conjugate poles

Let n be even number, $a_0 = 0$, $a_k = i e^{-1/\sqrt{n}}$, $k = 1, 2, \ldots, \frac{n}{2}$, $a_k = -i e^{-1/\sqrt{n}}$, $k = \frac{n}{2}+1, \frac{n}{2}+2, \ldots, n$. Then the graph of the function $\frac{\Lambda_{2n}(x)}{1+\ln n}$ for $n = 30$ is presented on the fig. 5 (axes mean the same as in the subsection «Two real poles»).

Behavior of the Lebesgue constant in this case is described by the fig. 6.

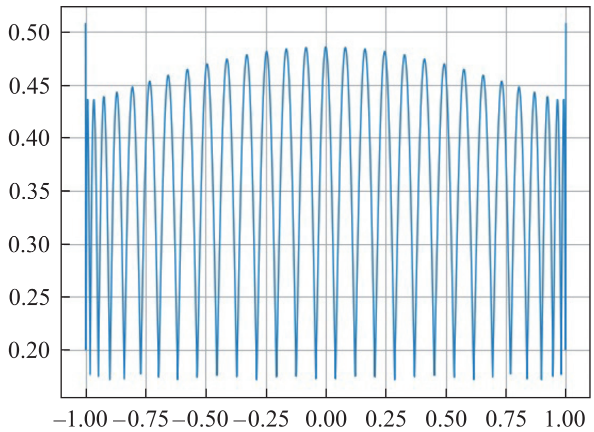

Fig. 5. Graph of $\frac{\Lambda_{2n}(x)}{1+\ln n}$ for the case of complex conjugate poles and $n = 30$

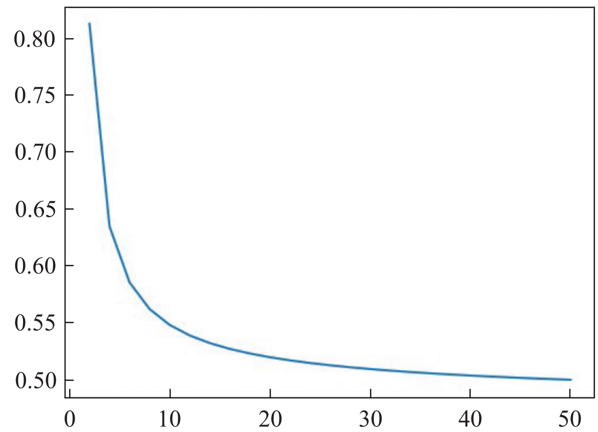

Fig. 6. Lebesgue constant for the case of complex conjugate poles

The results of all conducted experiments show, that the Lebesgue constant grows with logarithmic rate and allow us to make an assumption of independence of the estimation of the Lebesgue constant from the choice of zeroes of Chebyshev – Markov rational cosine fractions.

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