УДК 519.2

ПОСЛЕДОВАТЕЛЬНАЯ СТАТИСТИЧЕСКАЯ ПРОВЕРКА ГИПОТЕЗ ДЛЯ ДИСКРЕТНЫХ СЛУЧАЙНЫХ ДАННЫХ С БЛОЧНОЙ СТРУКТУРОЙ

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Аннотация. Рассмотрена актуальная математическая задача компьютерного анализа данных – задача последовательной статистической проверки простых гипотез о распределении вероятностей дискретных случайных данных, имеющих блочную структуру. Выведены легко интерпретируемые и удобные для компьютерной реализации явные выражения статистик последовательных тестов (статистических критериев). Разработан подход для вычисления характеристик эффективности решающих правил – вероятностей ошибочных решений и математических ожиданий случайного числа наблюдений, необходимых для обеспечения требуемой точности решений. Получены асимптотические разложения для указанных характеристик эффективности при искажении гипотетической вероятностной модели.

Ключевые слова: случайные дискретные данные; блочная структура; простые гипотезы; последовательный статистический тест; вероятность ошибки; математическое ожидание случайного числа наблюдений; асимптотические разложения.

SEQUENTIAL STATISTICAL HYPOTHESES TESTING FOR DISCRETE RANDOM DATA WITH BLOCK STRUCTURE

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Abstract. An important mathematical problem of computer data analysis – the problem of sequential statistical testing of simple hypotheses on probability distributions of discrete random data with a block structure – is considered. Explicit expressions of the sequential tests statistics, transparent for interpretation and convenient for computer realisation, are derived. An approach to calculate the performance characteristics – error probabilities of the decisions and mathematical expectations of the random number of observations required to guarantee the requested accuracy for decision rules – is

Образец цитирования:
Харин АЮ. Последовательная статистическая проверка гипотез для дискретных случайных данных с блочной структурой. *Журнал Белорусского государственного университета. Математика. Информатика.* 2026;1:75–80 (на англ.).
EDN: OPJJSF

For citation:
Kharin AYu. Sequential statistical hypotheses testing for discrete random data with block structure. *Journal of the Belarusian State University. Mathematics and Informatics.* 2026;1:75–80.
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developed. Asymptotic expansions for the mentioned performance characteristics under distortion of the hypothetical probability model are constructed.

Keywords: discrete random data; block structure; simple hypotheses; sequential statistical test; error probability; mathematical expectation of the random number of observations; asymptotic expansions.

Introduction

Data is one of the most essential resources of the world economy, and computer data analysis becomes a standard part of the modern life. Efficiency of data analysis defines the success in a growing variety of directions. Discrete data become an extremely important class of data for several reasons. Firstly, discrete data are natural for human being thinking. Secondly, they include binary data that describe many situations in terms of presence – absence, positive – negative, etc. Thirdly, discrete data can be used for description of a significantly rich family of observations and are more clearly processed by computers. Traditional methods of statistical analysis and software are often not applied to such data, as their assumptions are usually not satisfied for the discrete data models, or they are not effective.

As deterministic approach has a limited potential to describe the real-life processes, observed data is usually considered to be random, and probabilistic models are used if mathematics is needed. In these models, an important problem that often appears, is a problem of discrimination between two typical situations on the probability distributions of random data. These two typical situations can be formulated in terms of simple hypotheses on the probability distribution, and the problem turns to the problem of statistical testing of two simple hypotheses [1].

In many fields, especially in statistical quality control, automatic warning systems, personalised medicine and financial decision making, it is important to use the minimal number of observations that guarantee the requested accuracy [2]. Sequential statistical tests [3] are constructed in agreement to this principle with the assumption that the number of observations to be used is defined through the observation process, depend on observations themselves, and thus is a random variable. Due to the complex structure of sequential decision rules, usually analysis of their performance characteristics (error probabilities of decisions and mathematical expectations of the random number of required observations) is moved from theoretical one to the computer simulation [4].

In the paper for the important model of discrete data that has a block structure we develop an approach to calculate and analyse the performance characteristics of sequential tests. In practice the factual probability distribution of data often deviates from the hypothetical one – the hypothetical probability distribution is distorted [5; 6], so we also provide a possibility to calculate the characteristics under distortions of the hypothetical data model.

Sequential probability ratio test for discrete random data with block structure

Denote by $\mathbf{N}$, $\mathbf{Z}$, $\mathbf{R}$ correspondently sets of positive integer, integer and real numbers; $\mathbf{Z}_+ = \mathbf{N} \cup \{0\}$. Let independent identically distributed $K\tau$-dimensional discrete random vectors with a block structure be observed:

$$X_t = \left(x_{t,i}\right) = \begin{pmatrix} X_t^1 \\ X_t^2 \\ \vdots \\ X_t^K \end{pmatrix} \in A^{K\tau},\ X_t^k = \begin{pmatrix} x_{t,(k-1)\tau+1} \\ \vdots \\ x_{t,k\tau} \end{pmatrix} \in A^{\tau},\ k = 1, 2, \ldots, K,\ t \in \mathbf{N}, \tag{1}$$

where $A = \{a_1, \ldots, a_M\}$ is a finite set with $2 \le M < +\infty$ elements $a_1 < a_2 < \ldots < a_M$; X_t^k is the block number k at the moment t; $2 \le K < +\infty$ is a number of blocks; $\tau \in \mathbf{N}$ is a size of the blocks. Let us mention that the equal size for all blocks is set here just to simplify the notation of the results, but for the general case where blocks in the vector X_t may have different sizes the results derivation scheme remains valid.

The blocks $X_t^1, \ldots, X_t^K$ are assumed to be independent but the components inside them are stochastically dependent. Because of the block independence in equalities (1), the probability distribution of an observation X_t is separable:

$$P(U) ::= P\{X_t = U\} = \prod_{k=1}^{K} P^{(k)}\left(U^k\right), \tag{2}$$

where $P^k\left(U^k\right) ::= P\left\{X_t^k = U^k\right\}$ is the probability distribution of the random subvector X_t^k.

Assume that there are two alternatives on the probability distribution (2) in the following parsimonious parametric form:

$$P_\nu^k\left(U^k\right)=P^k\left(U^k;\nu\right)=a^{-J_\nu^k\left(U^k\right)},\ U^k\in A^\tau,\ k=1,\ 2,\ \ldots,\ K,\ \nu\in\{0,1\},\tag{3}$$

where $a\in\mathbf{R}$, $a>1$, is some parameter; $J_\nu^k\left(U^k\right):A^\tau\times\{0,1\}\to\mathbf{Z}_+$ is a function that satisfies the normalising condition

$$\sum_{U^k\in A^\tau}a^{-J_\nu^k\left(U^k\right)}=1.\tag{4}$$

Denote $J_\nu(U)=\sum_{k=1}^{K}J_\nu^k\left(U^k\right)$ and $P^\nu(U)=a^{-J_\nu(U)}$, $U\in A^{K\tau}$. There are two simple hypotheses considered on the parameters $\nu\in\{0,1\}$ of probability distribution (3):

$$H_0\colon \nu=0,\ H_1\colon \nu=1.\tag{5}$$

Let us denote the following accumulated log-likelihood ratio test statistic for the observed sample of size n:

$$\Lambda_n=\Lambda_n\left(X_1,\ \ldots,\ X_n\right)=\sum_{t=1}^{n}\lambda_t,\tag{6}$$

where

$$\lambda_t=\lambda\left(X_t\right)=\log_a\left(\frac{P_1\left(X_t\right)}{P_0\left(X_t\right)}\right)=\sum_{k=1}^{K}\log_a\left(\frac{P_1^k\left(X_t^k\right)}{P_0^k\left(X_t^k\right)}\right)\tag{7}$$

is the log-likelihood ratio for the observation vector X_t, $t\in\mathbf{N}$.

Denote by $\delta_{i,\,j}$ the Kronecker delta, by

$$n^k\left(U^k\right)=\sum_{t=1}^{n}\delta_{X_t^k,\,U^k},\ U^k\in A^\tau,$$

the number of the subvectors X_t^k observed within the sample $\{X_1,\ \ldots,\ X_n\}$ that are equal to U^k,

$$\sum_{U^k\in A^\tau}n^k\left(U^k\right)\equiv n,\ k=1,\ 2,\ \ldots,\ K.$$

Theorem 1. *For the model (1)–(4) the sequence of statistics (6) and (7) for hypotheses (5) is a homogeneous Markov chain with discrete time, and it is represented in the form*

$$\Lambda_n=\Lambda_n\left(X_1,\ \ldots,\ X_n\right)=\sum_{k=1}^{K}\sum_{U^k\in A^\tau}n^k\left(U^k\right)\left(J_0^k\left(U^k\right)-J_1^k\left(U^k\right)\right)\in\mathbf{Z},\ n\in\mathbf{N}.\tag{8}$$

P r o o f. The Markov property [7] of the random sequence (6) follows from the independence of its increments (7) due to the independence of random observations (1). Representation (8) is derived by equivalent transformations of sequence of statistics (6) and (7) under assumptions (2)–(4).

Corollary. *If under theorem 1 conditions $M=2$, $K=1$ ($\tau=1$, $A=\{0,1\}$), then test statistic (6) is*

$$\Lambda_n=\Lambda_n\left(x_1,\ \ldots,\ x_n\right)=n_0\left(J_0(0)-J_1(0)\right)+\left(n-n_0\right)\left(J_0(1)-J_1(1)\right),\ n\in\mathbf{N},$$

where n_0 denotes the number of the observations equal to 0.

Using statistics (8), we construct the sequential probability ratio test [3] for hypotheses (5) as follows. The decision after n observations obtained ($n=1,\ 2,\ \ldots$) is

$$d=d(n)=\mathbf{1}_{[C_+,\,+\infty)}\left(\Lambda_n\right)+2\cdot\mathbf{1}_{(C_-,\,C_+)}\left(\Lambda_n\right),\tag{9}$$

where $\mathbf{1}_D(\cdot)$ denotes the indicator function of a set D. Decisions $d=0$ and $d=1$ mean the observation process termination and acceptance of correspondently H_0 or H_1 after n dicrete random vectors observed; decision $d=2$ means that the $(n+1)$ vector should be observed; C_-, $C_+\in\mathbf{Z}$, $C_-<C_+$ are parameters of the decision rule (8) called thresholds; in practice they are often calculated according to [3]:

$$C_-=\left[\log_a\left(\frac{\beta_0}{1-\alpha_0}\right)\right],\ C_+=\left[\log_a\left(\frac{1-\beta_0}{\alpha_0}\right)\right],$$

where α_0, β_0 are the admissible values of error type I (to accept H_1, when H_0 is true) and type II (H_1 is true, H_0 is accepted) probabilities; $[\cdot]$ means the integer part of an argument.

Performance characteristics of the sequential probability ratio test (9)

With thresholds (9) the factual values of error probabilities of type I and II may differ from α_0, β_0, and the problem of the factual values calculation of the performance characteristics is open for the sequential tests [6; 8–10].

Introduce the notations: $\mathbf{I}_k$ for the identity matrix of size k; $\mathbf{0}_{m \times n}$ for the zero-matrix of size $(m \times n)$; $\mathbf{1}(\cdot)$ for the unit step function; $\mathbf{1}_k$ for the k-vector column with components equal 1. Denote by $t^{(\nu)}$ the expected value of the random number of observations (sample size) provided the true hypothesis is H_ν, $\nu \in \{0, 1\}$, and by α, β the factual values of error type I and II probabilities for test (9). Also, let $N = C_+ - C_-$; $P^{(\nu)} = \left(p_{ij}^{(\nu)}\right) = \begin{pmatrix} \mathbf{I}_2 & \vdots & \mathbf{0}_{2 \times N} \\ --- & \vdots & --- \\ R^{(\nu)} & \vdots & Q^{(\nu)} \end{pmatrix}$ be the matrix of size $(N+2) \times (N+2)$ with blocks $R^{(\nu)}$, $Q^{(\nu)}$ defined by

$$p_{ij}^{(\nu)} = \begin{cases} \sum\limits_{U \in A^{K\tau}} \delta_{J_0(U) - J_1(U),\, j-i} P^\nu(U),\ i, j \in (C_-, C_+), \\ \sum\limits_{U \in A^{K\tau}} \mathbf{1}\left(C_- - i + J_1(U) - J_0(U)\right) P^\nu(U),\ i \in (C_-, C_+),\ j = C_-, \\ \sum\limits_{U \in A^{K\tau}} \mathbf{1}\left(J_0(U) - J_1(U) + i - C_+\right) P^\nu(U),\ i \in (C_-, C_+),\ j = C_+. \end{cases}$$

Denote

$$\pi^{(\nu)} = \left(\pi_i^{(\nu)}\right),\ \pi_i^{(\nu)} = \sum_{U \in A^{K\tau}} \delta_{J_0(U) - J_1(U),\, i} P^\nu(U),\ i \in \{C_- + 1, \ldots, C_+ - 1\},$$

$$\pi_{C_+}^{(\nu)} = \sum_{i \geq C_+} \sum_{U \in A^{K\tau}} \delta_{J_0(U) - J_1(U),\, i} P^\nu(U),$$

$$\pi_{C_-}^{(\nu)} = \sum_{i \leq C_-} \sum_{U \in A^{K\tau}} \delta_{J_0(U) - J_1(U),\, i} P^\nu(U),$$

$$S^{(\nu)} = \mathbf{I}_N - Q^{(\nu)},\ B^{(\nu)} = \left(S^{(\nu)}\right)^{-1} R^{(\nu)},\ \nu \in \{0, 1\},$$

and let $W_{(i)}$ means the i column of the matrix W.

Theorem 2. *If under the model (1)–(5) $\left|S^{(\nu)}\right| \neq 0$, $\nu \in \{0, 1\}$, then the performance characteristics of sequential decision rule (9) are calculated in the explicit form*

$$t^{(\nu)} = \left(\pi^{(\nu)}\right)' \left(S^{(\nu)}\right)^{-1} \mathbf{1}_N + 1,\ \alpha = \left(\pi^{(0)}\right)' B_{(2)}^{(0)} + \pi_{C_+}^{(0)},\ \beta = \left(\pi^{(1)}\right)' B_{(1)}^{(1)} + \pi_{C_-}^{(1)}.$$

P r o o f. Proof is based on the theory of discrete time finite homogeneous Markov chains. The sequence

$$\zeta_n = C_- \cdot \mathbf{1}_{(-\infty, C_-]}(\Lambda_n) + C_+ \cdot \mathbf{1}_{[C_+, +\infty)}(\Lambda_n) + \Lambda_n \cdot \mathbf{1}_{(C_-, C_+)}(\Lambda_n)$$

is a homogeneous Markov chain with $N + 2$ states, and C_-, C_+ are the absorbing states.

Case of distorted probability distributions $\left\{\boldsymbol{P}_\nu^k(\cdot)\right\}$

Let $\bar{P}_\nu(U)$ be the factual probability distribution of the independent observations $X_1, X_2, \ldots \in A^{K\tau}$ that may differ from the hypothetical probability distribution $P_\nu(U)$ determined by probability distribution (3) due to some model distortion.

Denote the sequential statistical test of the form (6) and (9) based on the function

$$\lambda(U) = \log_a\left(\frac{P_1(U)}{P_0(U)}\right), \; U \in A^{K\tau},$$

by $\delta_\lambda = (\tau_\lambda, d_\lambda)$, where $\tau_\lambda = \inf\{n : d_\lambda(n) \in \{0, 1\}\}$ is the random stopping moment of the test (random number of observations used to accept one of hypotheses (5)). Let for this test

$$\alpha = \alpha(\delta_\lambda) = E_0\{\bar{P}_0\{d_\lambda = 1 | \tau_\lambda\}\}, \; \beta = \beta(\delta_\lambda) = E_1\{\bar{P}_1\{d_\lambda = 0 | \tau_\lambda\}\} \tag{10}$$

be the factual values of the error type I and type II probabilities, respectively, where $E_\nu\{\cdot\}$ means mathematical expectation with the probability distribution $\bar{P}_\nu$;

$$t^{(\nu)} = t^{(\nu)}(\delta_\lambda) = E_\nu\{\tau_\lambda\}, \; \nu \in \{0, 1\}, \tag{11}$$

is the conditional mathematical expectation of the random size of the sample τ_λ under the true hypothesis is H_k.

Consider the set of performance characteristics $\{\alpha(\delta_\lambda), \beta(\delta_\lambda), t_0(\delta_\lambda), t_1(\delta_\lambda)\}$ for the test δ_λ. Theorem 2 gives expressions for the performance characteristics (10) and (11) of the test δ_λ in case, where the function $\lambda(\cdot)$ defined by the probability distribution (3) and the sequence of statistics (7) has a discrete set of values, and there are no distortions ($\bar{P}_\nu \equiv P_\nu$). In the general case for the test δ_λ introduce the notations ($\nu = 0, 1$)

$$Q^{(\nu)} = \left(q_{ij}^{(\nu)}\right), \; q_{ij}^{(\nu)} = \bar{P}_\nu\{\lambda(X_1) = j - i\}, \; i, j \in \{C_- + 1, \ldots, C_+ - 1\},$$

$$R^{(\nu)} = \left(r_{ij}^{(\nu)}\right), \; i \in \{C_- + 1, \ldots, C_+ - 1\}, \; j = C_-, C_+,$$

$$r_{iC_-}^{(\nu)} = \bar{P}_\nu\{\lambda(X_1) \le C_- - i\}, \; r_{iC_+}^{(\nu)} = \bar{P}_\nu\{\lambda(X_1) \ge C_+ - i\},$$

$$\pi^{(\nu)} = \left(\pi_i^{(\nu)}\right), \; \pi_i^{(\nu)} = \bar{P}_\nu\{\lambda(X_1) = i\}, \; i \in \{C_- + 1, \ldots, C_+ - 1\},$$

$$\pi_{C_-}^{(\nu)} = \bar{P}_\nu\{\lambda(X_1) \le C_-\}, \; \pi_{C_+}^{(\nu)} = \bar{P}_\nu\{\lambda(X_1) \ge C_+\}.$$

As it follows from the finite Markov chains theory [7], the condition $\left|S^{(\nu)}\right| \ne 0$ is equivalent to the test finiteness: $\bar{P}_\nu\{\tau_\lambda < \infty\} = 1, \; \nu \in \{0, 1\}$. This condition is satisfied [2] for case of independent identically distributed random observations, except the case, where $\bar{P}_\nu\{\lambda(X_1) = 0\} = 1$ (in this case $P_0(\cdot) = P_1(\cdot)$).

Let $\underline{\lambda}(U), \bar{\lambda}(U) : A^{K\tau} \to \mathbf{R}$ be some functions, $\delta_{\underline{\lambda}}, \delta_{\bar{\lambda}}$ be the sequential tests based on these functions, $\underline{\Lambda}_n = \sum_{t=1}^{n} \underline{\lambda}(X_t), \; \bar{\Lambda}_n = \sum_{t=1}^{n} \underline{\lambda}(X_t)$. Denote the Markov stopping moments

$$s_0 = \inf\{n : \Lambda_n \le C_-\}, \; s_1 = \inf\{n : \Lambda_n \ge C_+\}, \; \underline{s}_0 = \inf\{n : \underline{\Lambda}_n \le C_-\},$$

$$\underline{s}_1 = \inf\{n : \underline{\Lambda}_n \ge C_+\}, \; \bar{s}_0 = \inf\{n : \bar{\Lambda}_n \le C_-\}, \; \bar{s}_1 = \inf\{n : \bar{\Lambda}_n \ge C_+\}.$$

Denote also

$$\bar{\alpha} = \alpha(\delta_{\bar{\lambda}}), \; \bar{\beta} = \beta(\delta_{\bar{\lambda}}), \; \underline{\alpha} = \alpha(\delta_{\bar{\lambda}}), \; \underline{\beta} = \beta(\delta_{\bar{\lambda}}),$$

$$Y = \{U \in A^{K\tau} : \lambda(X) \in (C_- - C_+, \; C_+ - C_-)\}.$$

Let $t^{(\nu)}$ is used for the mathematical expectation of the random number of observations of the sequential test based on the function $\lambda(\cdot)$ under the true hypothesis H_ν, before stopping and accepting one of the hypothesis (5). Also, let $t^{(\nu)}(\delta_{\bar{\lambda}})$ and $t^{(\nu)}(\delta_{\underline{\lambda}})$ are used for similar characteristics for the sequential tests based on functions $\bar{\lambda}(\cdot)$ and $\underline{\lambda}(\cdot)$, respectively.

Theorem 3. *Let for the sequential statistical test defined above the functions* $\underline{\lambda}(\cdot), \bar{\lambda}(\cdot)$ *satisfy the conditions*

$$\underline{\lambda}(U) \le \lambda(U) \le \bar{\lambda}(U) \;\; \forall U \in Y,$$

and

$$t^{(\nu)} < \infty, \; t^{(\nu)}(\delta_{\bar{\lambda}}) < \infty, \; t^{(\nu)}(\delta_{\underline{\lambda}}) < \infty, \; \nu = 0, 1.$$

Then

$$\underline{\alpha} \le \alpha \le \overline{\alpha},\ \overline{\beta} \le \beta \le \underline{\beta}$$

and in the asymptotics $\overline{\alpha} - \underline{\alpha} \to 0,\ \underline{\beta} - \overline{\beta} \to 0$ *the following results are valid:*

$$t_{-}^{(\nu)} \le t^{(\nu)} \le t_{+}^{(\nu)},\ \nu = 0, 1,\ t_{\pm}^{(\nu)} = T_{\pm}^{(\nu)} + o(1),$$

where

$$T_{-}^{(0)} = \overline{\alpha} E_0 \left\{ \overline{s}_1 \mid \overline{s}_1 < \overline{s}_0 \right\} + \left(1 - \underline{\alpha}\right) E_0 \left\{ \underline{s}_0 \mid \underline{s}_0 < \underline{s}_1 \right\},$$

$$T_{+}^{(0)} = \underline{\alpha} E_0 \left\{ \underline{s}_1 \mid \underline{s}_1 < \underline{s}_0 \right\} + \left(1 - \overline{\alpha}\right) E_0 \left\{ \overline{s}_0 \mid \overline{s}_0 < \overline{s}_1 \right\},$$

$$T_{-}^{(1)} = \underline{\beta} E_1 \left\{ \underline{s}_0 \mid \underline{s}_0 < \underline{s}_1 \right\} + \left(1 - \overline{\beta}\right) E_0 \left\{ \overline{s}_1 \mid \overline{s}_1 < \overline{s}_0 \right\},$$

$$T_{+}^{(1)} = \overline{\beta} E_1 \left\{ \overline{s}_0 \mid \overline{s}_0 < \overline{s}_1 \right\} + \left(1 - \underline{\beta}\right) E_1 \left\{ \underline{s}_1 \mid \underline{s}_1 < \underline{s}_0 \right\}.$$

P r o o f. Proof is based on analysis of relations for the introduced Markov moments and for the random events that are generated by them.

Conclusions

An approach to calculate and analyse the performance characteristics of sequential tests for discrete random data is proposed. The block structure in the data makes it possible to consider data not entire observation after entire observation but define the stopping moment on the basis of information available in some blocks only. The results can be applied to construct robust sequential tests [8] for discrete random data and for the model of random sequences with a trend [9]. The results are also potentially applicable to the case of more than two hypotheses [11], complex hypotheses [10] and to the analysis of truncated sequential tests [12].

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Received 24.12.2025 / revised 19.01.2026 / accepted 29.01.2026.