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ДВУХСЛОЙНЫЕ ГРАФЕНОВЫЕ ГЕТЕРОСТРУКТУРЫ ДЛЯ ЗАМЕДЛЕНИЯ ВОЛН: ОПЕРАТОРНЫЙ МЕТОД В ПРИМЕНЕНИИ К РЕШЕНИЮ ВОЛНОВОДНОЙ ЗАДАЧИ

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Замедление фазовой скорости света в среде имеет различные применения. Среди них – генерация электромагнитного излучения с использованием когерентного черенковского механизма. В то же время существует потребность в компактных источниках терагерцового излучения. Благодаря уникальным свойствам графена гетероструктуры, состоящие из чередующихся слоев графен/диэлектрик, могут работать в качестве среды для этой цели. При помощи операторного метода получены коэффициенты замедления и распространения для мод, поддерживаемых в двухслойной графеновой структуре. Операторный подход позволяет использовать бескоординатные обозначения и, следовательно, работать со сколь угодно сложными гетероструктурами (включающими, например, анизотропные слои). Изучена зависимость степени замедления волн в графеновых сэндвич-структурах от межслойного расстояния и величины химического потенциала графена. Полученные результаты открывают перспективы для создания новых источников терагерцового излучения.

Ключевые слова: графен; гетероструктура; излучение Вавилова – Черенкова; операторный метод; фазовая скорость.

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TWO LAYER GRAPHENE HETEROSTRUCTURES FOR WAVES SLOWING DOWN: OPERATOR APPROACH TO WAVEGUIDE PROBLEM

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Slowing down the phase velocity of light in media has various applications. The generation of electromagnetic radiation using coherent Cherenkov mechanism is among them. Meanwhile, there is a need for compact terahertz radiation sources. Due to outstanding graphene properties, heterostructures consisting of alternating graphene/dielectric layers can operate as a medium for the generation of terahertz radiation. In the present paper, the slowing down and propagation coefficients for the modes supported in a two-layer graphene structure are studied. The study is conducted by means of the operator approach to wave propagation in stratified structures. The operator approach allows one to use coordinates-free notations and to consider consequently arbitrarily complex heterostructures (including anisotropic layers, for instance). The influence of interlayer distance and the value of graphene chemical potential on waves slowdown is determined. The obtained results open up prospects for creating a new type of terahertz radiation sources.

Keywords: graphene; heterostructure; Cherenkov radiation; operator approach; phase velocity.

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Introduction

Nowadays, one of the challenges in applied electromagnetics is to find new approaches to controlling and, in particular, slowing down the phase velocity of light in media [1]. Potential applications of waves slowdown are ubiquitous and include communication, nonlinear optical amplification, optical data processing and temporary light storage [2]. Additionally, the effect of electromagnetic waves slowdown can be used to implement generators of electromagnetic radiation, working on the principle of the radiative instability of a directional electron beam [3; 4]. The slowdown of an electromagnetic wave can be applied for the generation of coherent Cherenkov radiation in the terahertz frequency range [5]. In its turn, the terahertz frequency range is in demand for many scientific and technical applications [6]. Among them, there is a need for low-cost sources for this frequency range [6].

Although promising results have been obtained on the light slowdown in solids and semiconductor-based nanostructures operating at room temperature, it still remains a challenge to incorporate such schemes in optoelectronic devices [7; 8]. Therefore, there is a great interest in alternative approaches using photonic structures, such as metallic gratings [9], microcavities [10], photonic crystals [11], semiconductor waveguide ring resonators [12; 13], etc.

Graphene and graphene-based structures (such as carbon nanotubes) are attractive for many functional materials due to their strong interaction with the external field, relatively low ohmic losses, and tunability [1; 14]. Determining peculiar properties of transport of charge carriers, the band structure is the reason for unique graphene characteristics [15]. The unusual low-energy electronic structure includes its conduction and valence bands that meet at the Dirac points [15; 16]. Graphene supports an extraordinarily strong electric current (with the density on the order of 10^8 A/cm^2) without experiencing degradation [17] and reveals the ballistic length of electrons propagation on a macroscopic scale (about 28 μm according to [18]). The size of graphene-based structures is of a nanometer-scale at least in one direction and they can support surface plasmon waves [19]. It was shown that surface electromagnetic waves can be slowed down significantly in carbon nanotubes and graphene [19; 20] and thus provide better conditions for the synchronization of an electron beam and a surface electromagnetic wave [5]. Additionally, π-electrons of graphene can coherently radiate from macroscopic ballistic length [5].

For the application to absorbers, in reference [14] it was demonstrated that graphene sandwich structures consisting of alternating layers of graphene and dielectric spacer exhibit advantageous features compared to a single graphene layer [14; 21]. Furthermore, such heterostructures suggest more means to perform electromagnetic tuning [5; 21]. Two graphene layers in vacuum demonstrate the remarkable effect of the slowdown in the terahertz frequency range [5], but these structures have not been studied in the presence of a dielectric spacer. This makes it interesting to consider the potential of graphene sandwich structures for the aims of a light slowdown.

Among the various methods of studying electromagnetic waves propagation in multilayer structures the most fruitful one is the operator formalism [22; 23]. Particularly, it allows one to find compact covariant formulas for very complex systems avoiding cumbersome calculations that arise in the process of adaptation of different coordinate systems for different layers [24; 25]. In what follows, the first demonstration of operator approach to study graphene sandwich structures is presented.

Methods

Operator approach. Operator approach was used to perform calculations. Operator formalism allows one to study properties of multilayer structures when separate layers are characterized by bulk parameters (such as permittivity and permeability). However, it is possible to extend the approach to multilayers including impedance sheets.

Monochromatic electromagnetic wave (angular frequency ω) propagating in a planar slab of a homogeneous medium is considered. The medium is characterized by a dielectric permittivity tensor $\hat{\varepsilon}$ and magnetic permeability tensor $\hat{\mu}$ by constitutive equations as

$$\boldsymbol{D} = \hat{\varepsilon}(\omega)\boldsymbol{E},\ \boldsymbol{B} = \hat{\mu}(\omega)\boldsymbol{H},$$

where $\boldsymbol{E}$, $\boldsymbol{H}$, $\boldsymbol{D}$, $\boldsymbol{B}$ are respectively the strengths of electric and magnetic fields, electric displacement, and magnetic induction. When one looks for the solution of Maxwell's partial differential equations in the form $\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}(z)\exp(ik_0\boldsymbol{br})$, they can be reduced to the four first order ordinary differential equations for the tangential field components $\boldsymbol{W} = (\boldsymbol{H_t},\ \boldsymbol{n}\times\boldsymbol{E})^{\mathrm{T}}$, where $\boldsymbol{b} = \dfrac{\boldsymbol{k_t}}{k_0}$ is the normalized tangential wavevector, $\boldsymbol{n}$ is the unit vector along z-axis, T denotes transpose operation, $\boldsymbol{H_t} = \hat{I}\boldsymbol{H} = (\boldsymbol{H_x},\ \boldsymbol{H_y})^{\mathrm{T}}$, and $\boldsymbol{n}\times\boldsymbol{E} = (-\boldsymbol{E_x},\ \boldsymbol{E_y})^{\mathrm{T}}$ ($\hat{I} = \hat{1} - \boldsymbol{n}\otimes\boldsymbol{n}$ is the projector onto the xOy plane). Thus, the system of four differential equations reads

$$\frac{d\boldsymbol{W}(z)}{dz} = ik_0\hat{M}\boldsymbol{W}(z) = ik_0\begin{pmatrix} A & B \\ C & D \end{pmatrix}\boldsymbol{W}(z). \tag{1}$$

Where 4×4 matrix $\hat{M}$ is a block matrix, whose 2×2 blocks $\hat{A}$, $\hat{B}$, $\hat{C}$ and $\hat{D}$ can be found in [22]. Fundamental solution of equation (1) for a homogeneous bianisotropic medium with constant matrix $\hat{M}$ is the matrix exponential [22; 26]

$$\boldsymbol{W}(z) = \exp\left(ik_0 z\hat{M}\right)\boldsymbol{W}(0),$$

where $\boldsymbol{W}(0)$ is the initial field at $z = 0$. Then, the result of the transmission through a stack of N slabs can be expressed as follows

$$\boldsymbol{W}(T) = \prod_{j=1}^{N}\exp\left(ik_0 t_j\hat{M}_j\right)\boldsymbol{W}(0) = \hat{C}(T)\boldsymbol{W}(0),$$

where $T = \sum_{j=1}^{N} t_j$ is the thickness of the stack and $\hat{C}$ is the spatial evolution operator. It should be noted that the order of multiplication of non-commuting exponential operators is important.

Graphene can be considered as an impedance sheet, on which the electromagnetic field excites surface currents. It leads to the field discontinuity across the sheet. The field discontinuity is described by the boundary conditions. Then the fields $(\boldsymbol{H_{1t}},\ \boldsymbol{E_{1t}})$ right before the impedance sheet and right after it $(\boldsymbol{H_{2t}},\ \boldsymbol{E_{2t}})$ can be related by means of a spatial evolution operator $\hat{C}_\sigma$ of the impedance sheet characterized by conductivity σ

$$\begin{pmatrix} \boldsymbol{H_{2t}} \\ \boldsymbol{n}\times\boldsymbol{E_2} \end{pmatrix} = \hat{C}_\sigma\begin{pmatrix} \boldsymbol{H_{1t}} \\ \boldsymbol{n}\times\boldsymbol{E_1} \end{pmatrix},\ \hat{C}_\sigma = \begin{pmatrix} \hat{I} & -\dfrac{4\pi}{c}\sigma\hat{I} \\ \hat{0} & \hat{I} \end{pmatrix}.$$

If σ is the graphene's conductivity, then $\hat{C}_\sigma$ is the spatial evolution operator of a graphene sheet. To describe a slab of isotropic dielectric with permittivity ε one uses the following M-operator

$$\hat{M}_d = \begin{pmatrix} 0 & 0 & \varepsilon - b^2 & 0 \\ 0 & 0 & 0 & \varepsilon \\ 1 & 0 & 0 & 0 \\ 0 & 1 - \dfrac{b^2}{\varepsilon} & 0 & 0 \end{pmatrix},$$

where b is a tangential component of wavevector divided by vacuum wavenumber (k_0). It has the meaning of the slowdown coefficient of electromagnetic wave in the problems of guided waves, i. e. $b = \frac{c}{v_{\mathrm{ph}}}$ (c is vacuum speed of light, v_{ph} is phase speed of guided wave). The matrix exponential of M-operator $\exp\left(ik_0\hat{M}_d t_s\right)$ of the dielectric slab provides it spatial evolution operator, t_s is the thickness of the slab.

Spatial evolution operator of a multilayer is a consequent product of evolution operators of constituents (the sequence in the product is opposite to the one of layers).

In the problem of guided waves, one has to find eigenmodes of the system. To find eigenmodes one solves the boundary problem when on the one side outside the multilayer there is a plane wave above (superscript 1) and on the other side – a plane wave below (superscript 2). These waves attenuate while retreating from the interface. The field right above and below the multilayer are related by means of the evolution operator corresponding to the multilayer $\hat{C}(T)$ (T stands for the total thickness of the multilayer)

$$\begin{pmatrix} \boldsymbol{H}_t^{(2)} \\ \hat{\gamma}_2 \boldsymbol{H}_t^{(2)} \end{pmatrix} = \hat{C}(T) \begin{pmatrix} \boldsymbol{H}_t^{(1)} \\ \hat{\gamma}_1 \boldsymbol{H}_t^{(1)} \end{pmatrix}, \tag{2}$$

where $\boldsymbol{H}_t^{(1,\,2)}$ are the tangential components of the magnetic field of waves above and below the multilayer, $\hat{\gamma}_{1,\,2}$ are the surface impedances of waves above and below. In case of TE and TM polarized waves surface impedance coincides with the wave impedances [22]

$$\hat{\gamma}_1 = -\frac{\sqrt{\varepsilon_{\mathrm{in}} - \boldsymbol{b}^2}}{\varepsilon_{\mathrm{in}}} \frac{\boldsymbol{a} \otimes \boldsymbol{a}}{\boldsymbol{b}^2} - \frac{1}{\sqrt{\varepsilon_{\mathrm{in}} - \boldsymbol{b}^2}} \frac{\boldsymbol{b} \otimes \boldsymbol{b}}{\boldsymbol{b}^2},$$

$$\hat{\gamma}_2 = \frac{\sqrt{\varepsilon_{\mathrm{out}} - \boldsymbol{b}^2}}{\varepsilon_{\mathrm{out}}} \frac{\boldsymbol{a} \otimes \boldsymbol{a}}{\boldsymbol{b}^2} + \frac{1}{\sqrt{\varepsilon_{\mathrm{out}} - \boldsymbol{b}^2}} \frac{\boldsymbol{b} \otimes \boldsymbol{b}}{\boldsymbol{b}^2},$$

where $\boldsymbol{a} = \boldsymbol{b} \times \boldsymbol{n}$, $\varepsilon_{\mathrm{in}}$ is the permittivity of the medium before the multilayer and $\varepsilon_{\mathrm{out}}$ is the one after the multilayer. In equation (2) one can easily get rid of fields above and below the structure and arrive at the eigenvector and the eigenvalue problem [22; 26], e. g.

$$\left(\hat{\gamma}_2, -\hat{I}\right)\hat{C}(T)\begin{pmatrix} \hat{I} \\ \hat{\gamma}_1 \end{pmatrix} \boldsymbol{H}_t^{(1)} = 0. \tag{3}$$

There is a nontrivial solution of the above equation only in the case when the determinant of the matrix equals zero, i. e.

$$\det\left[\left(\hat{\gamma}_2, -\hat{I}\right)\hat{C}(T)\begin{pmatrix} \hat{I} \\ \hat{\gamma}_1 \end{pmatrix}\right] = 0. \tag{4}$$

Thus, equation (4) is a necessary condition for the existence of guided modes. It relates the deceleration factor b of a guided mode with its angular frequency ω. Therefore, equation (4) is usually called as dispersion equation.

Graphene conductivity. The conductivity of a graphene monolayer in the terahertz range can be expressed by the intraband term [27]:

$$\sigma^{\text{intra}}\left(\omega,\ \mu,\ T,\ \Gamma_{\text{trans}}\right) = \frac{2e^2 k_B T}{\pi\hbar^2}\ln\left(2\cosh\left(\frac{\mu}{2k_B T}\right)\right)\frac{i}{\omega + i\Gamma_{\text{trans}}},$$

where μ – graphene's chemical potential; T – temperature; Γ_{trans} – intraband relaxation rate; e – electron charge; $\hbar$ – reduced Planck's constant; k_B – Boltzmann's constant. The intraband conductivity has the Drude form and depends mainly on the chemical potential [27] which can be changed by doping, for example. The influence of the graphene chemical potential through its conductivity on the slowing down of electromagnetic waves is demonstrated below. In the present paper the temperature value is defined as 300 K and the value of intraband relaxation rates is 0.03 THz [18], unless otherwise is specified.

Results and discussion

A freely suspended graphene sheet can support surface waves but its slowdown coefficient is not very high [1; 5; 28]. As an example, let us consider the graphene/dielectric/graphene sandwich (the permittivity of the dielectric is ε_d) (fig. 1). To simplify calculations, we assume that the structure is infinite in the longitudinal direction and surrounded by dielectric with ε_1 in the top part and with ε_2 in the bottom.

In case of using traditional Cherenkov or Smith – Purcell generation schemes, it is necessary to consider only the transverse-magnetic p-polarization [5]. Then, using equation (3), equation for eigenmodes of two layers graphene-dielectric system takes the form

$$i\tan\left(k_0 d\eta_d\right) = -\frac{\dfrac{\eta_d}{\varepsilon_d}\left(2Y - \dfrac{\varepsilon_1}{\eta_1} - \dfrac{\varepsilon_2}{\eta_2}\right)}{1 + \left(\dfrac{\eta_d}{\varepsilon_d}\right)^2\left(Y - \dfrac{\varepsilon_1}{\eta_1}\right)\left(Y - \dfrac{\varepsilon_2}{\eta_2}\right)}, \tag{5}$$

where $\eta_i = \sqrt{\varepsilon - b^2}$ is the normal component of the refractive vector in the corresponding medium, d is the dielectric spacer thickness, and $Y = \dfrac{4\pi\sigma}{c}$. This equation has a complicated form and in a general case it is possible to find the slowdown coefficient $\dfrac{c}{v_{\text{ph}}}$ only numerically. Two modes corresponding to synchronous and asynchronous oscillations are the solution of equation (5). Frequency of the asymmetric mode can be significantly less than one for single layer due to combination frequency effect. This leads to a much greater slowdown than it is possible with a single graphene monolayer (see [5]).

The value of the dielectric permittivity $\varepsilon_d = 2.6$ is chosen to correspond to PMMA [29], which is normally used in such graphene structures [14]. One of the key parameters affecting the slowdown in the sandwich structure is the distance between graphene. Figure 2, *a*, *b*, demonstrate the influence of dielectric thickness in the terahertz frequency region when the graphene chemical potential is 0.10 eV. The slowdown coefficient for asynchronous oscillations significantly exceeds that for synchronous oscillations. Such behavior arises from the fact that in the case of synchronous oscillations the whole sandwich structure interacts with electromagnetic wave like it does with a graphene monolayer.

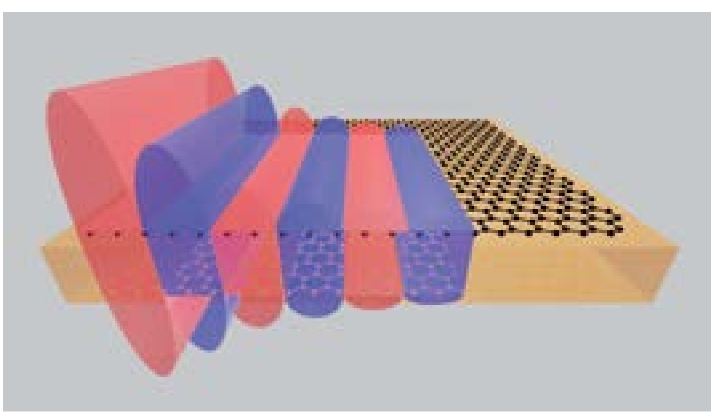

Fig. 1. Schematics of the considered system:
the dielectric spacer with ε_d in between of two graphene layers,
surrounded by a media with the permittivity $\varepsilon_1 = 1$.
A propagating eigenmode is pictured schematically

The maximum value of the slowdown coefficient in the considered system surpasses 200 and decreases with increasing distance between layers (fig. 2). This is caused by a weakening of the bond between the plasmon-polaritons that arise in the system [5]. At the same time, the change in the distance between the sheets has a weak effect on the propagation coefficient ξ. The ratio $\xi = \frac{\mathrm{Re}\, q}{\mathrm{Im}\, q}$ of the real and imaginary parts of the tangential component of wavevector $\boldsymbol{q}$ determines wave attenuation during propagation along the layers. If this ratio is close to unity, then the corresponding modes decay quickly. In other words, ξ determines the number of oscillations occurring before a wave dissipates in the system. The propagation coefficient strongly depends on graphene quality (fig. 3). In its turn, graphene quality characterized by the intraband relaxation rate Γ_{trans} which plays the main role in the terahertz region [27].

It was shown experimentally that the ballistic length in CVD graphene can exceed 28 μm [18] what corresponds to the relaxation rate parameter of about 0.03 THz. For a demonstration, in fig. 3 we show the propagation constant ξ for the two-layer graphene system (as in fig. 2) with a 10 nm dielectric spacer, when Γ_{trans} = 13 THz. The given value of the relaxation rate was demonstrated in [14; 21]. One can see that in this case the attenuation increases dramatically, and factually makes it impossible to use such a system for wave slowdown.

The dependence of the slowdown coefficient on the value of the graphene chemical potential μ in the case of asynchronous and synchronous oscillations is shown in fig. 4, *a*, *b*. The chemical potential has a significant effect on the value of the slowdown coefficient in graphene sandwiches. For a lower value of the chemical

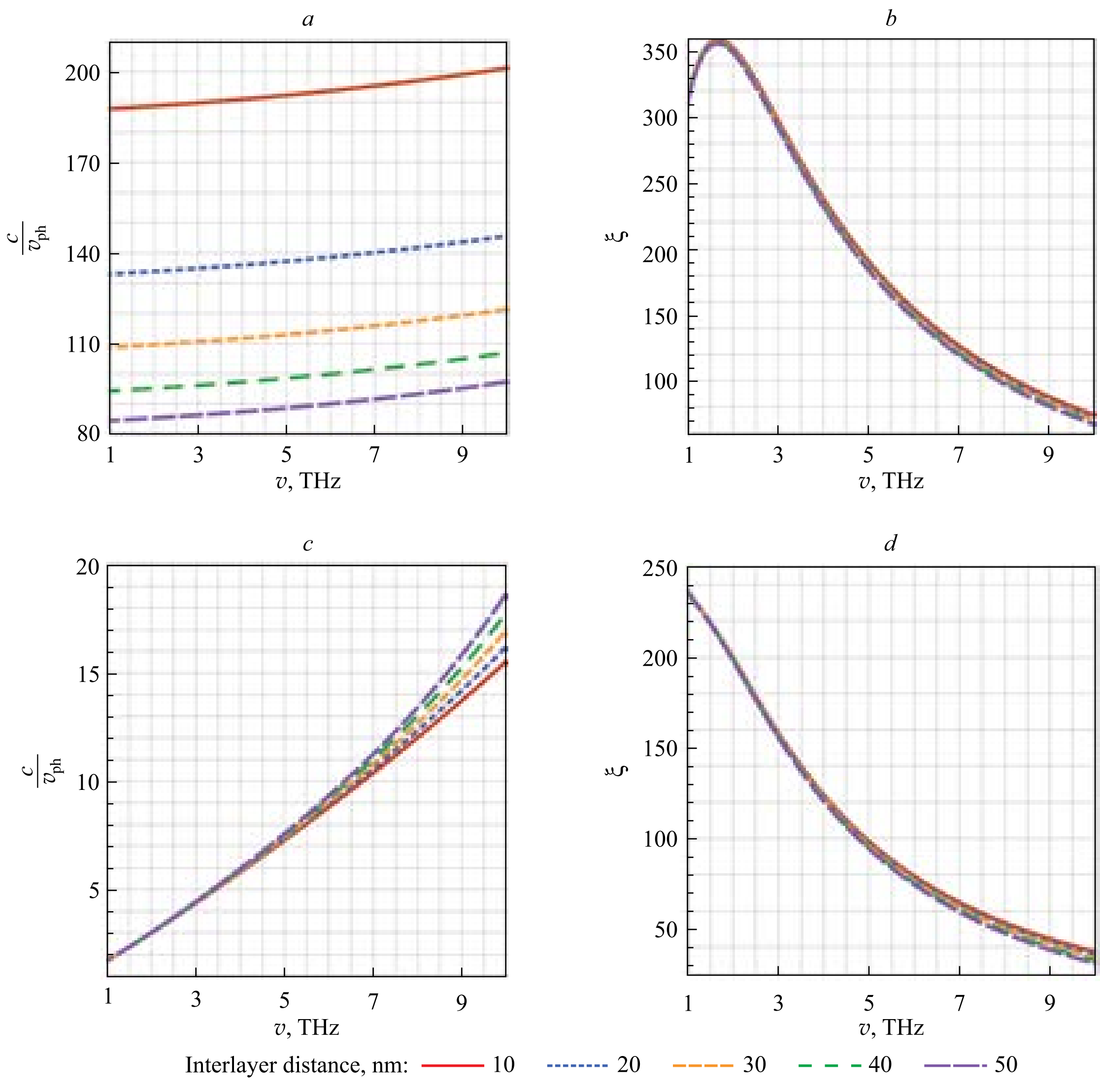

Fig. 2. The dependence of the slowdown coefficient on the frequency in the case (*a*) asynchronous and (*b*) synchronous oscillations for different interlayer distances when the graphene chemical potential is 0.10 eV; the value of the propagation coefficient ξ in the plane of the layers for the same (*c*) asynchronous and (*d*) synchronous oscillation

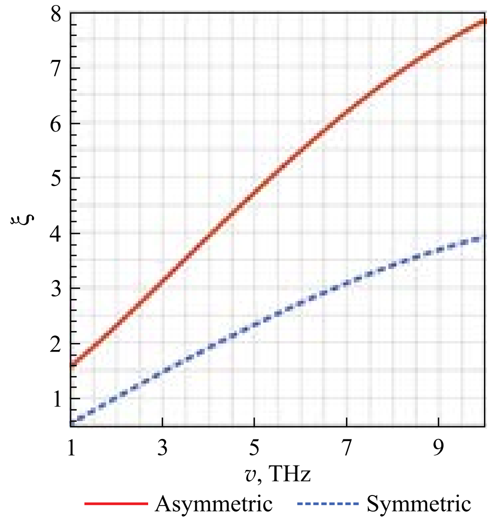

Fig. 3. The value of the propagation coefficient ξ in the plane of graphene layers with the dielectric spacer thickness 10 nm for Γ_{trans} = 13 THz

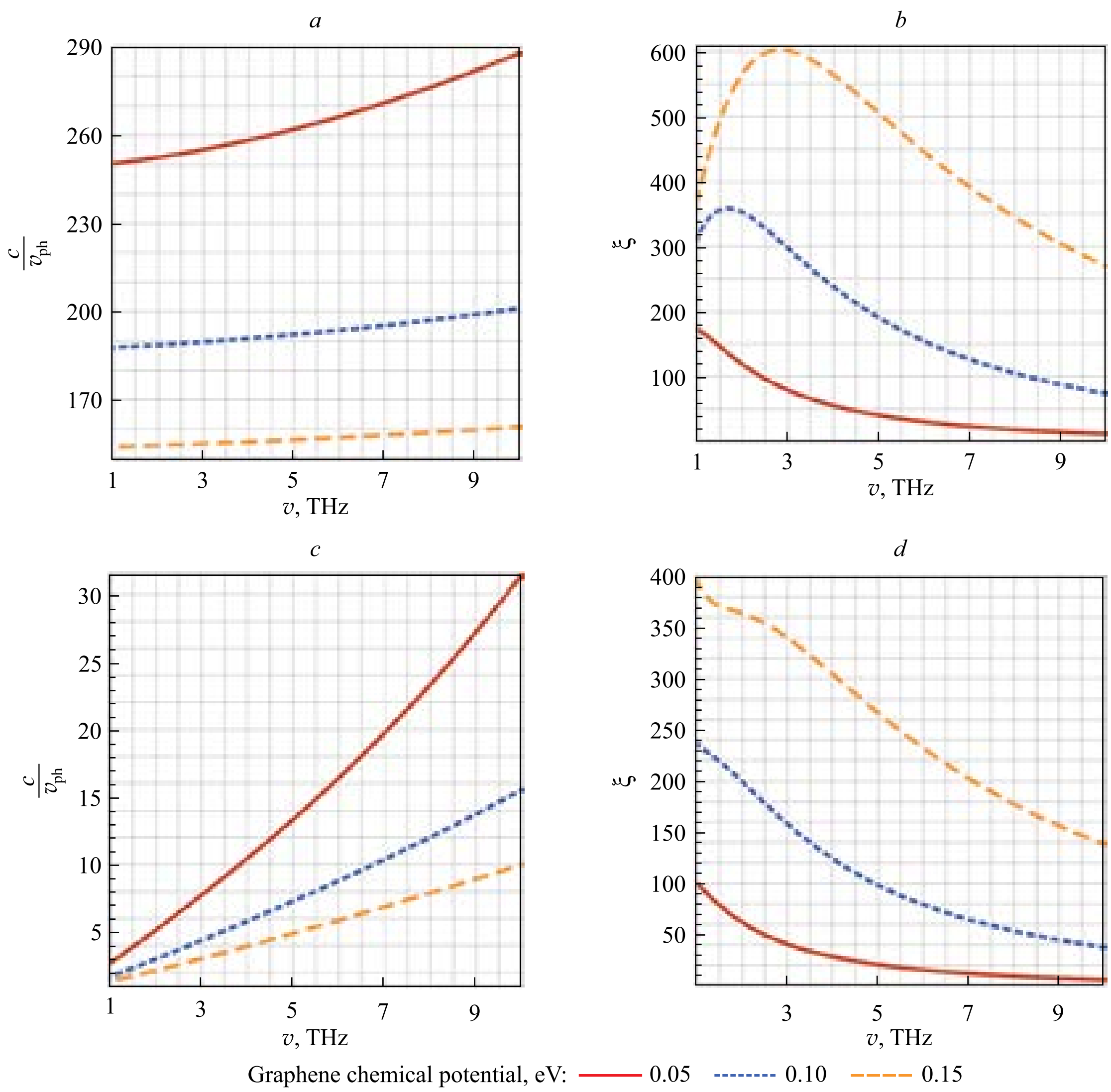

Fig. 4. The dependence of the slowdown coefficient on the frequency in the case (*a*) asynchronous and (*b*) synchronous oscillations for different values of the graphene chemical potential for the interlayer thickness 10 nm; the value of the propagation coefficient ξ in the plane of the layers for the same (*c*) asynchronous and (*d*) synchronous oscillation

potential, the slowdown coefficient almost reaches 300, which corresponds to the velocity of charge carriers in the graphene layer. Increasing of the graphene's chemical potential leads to the decrease of the light slowdown. This can be used in the dynamical tuning of graphene sandwiches, as μ value determines graphene's conductivity and can be changed by applying the electric field to graphene [5].

The propagation coefficient also significantly depends on the graphene chemical potential μ. Figure 4, *c*, *d*, show the dependence of the propagation coefficient ξ on the chemical potential. With the increase of μ, the propagation coefficient in the graphene sandwich increases as well. It can be explained by the change of the graphene conductivity.

Conclusion

Possessing unique physical properties, a pair of graphene layers combined with a dielectric allows one to obtain a structure with attractive physical properties. Graphene sandwich structures demonstrate a significant slowdown in the terahertz frequency range. In the case of a two graphene layers system, two eigenmodes appear: synchronous with slowdown coefficient close to one for single graphene layer, and asynchronous when the wave phase velocity almost reaches the value corresponding to the velocity of charge carriers. The value of this coefficient is determined by the interlayer distance and the value of graphene chemical potential, which can be tuned by electric voltage. At the same time, these modes decay rather slowly, which ensures the application of such structures in practice. The combination of such a system with an electron beam can be used to create a new generation of frequency-tunable compact terahertz sources.

To conclude, the application of the operator approach to graphene heterostructures has been demonstrated. The operator approach allows one to use coordinates-free notations and to consider consequently arbitrarily heterostructures. The results of the presented study open a way to the investigation of complex graphene-based multilayer systems.

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