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ОДНОЧАСТОТНЫЙ ЛАЗЕР НА БАЗЕ ЛЕГИРОВАННОГО ИОНАМИ Er – Yb ОПТИЧЕСКОГО ВОЛОКНА С ВОЛОКОННЫМИ БРЭГГОВСКИМИ РЕШЕТКАМИ

Дж. АЛЬРЕМЕИТИ[1), 2)]***, И. А. ГОНЧАРЕНКО***[2), 3)]

[1)]*Институт технологических инноваций, 9639, Масдар-Сити, Абу-Даби, Объединенные Арабские Эмираты*
[2)]*Белорусский государственный университет, пр. Независимости, 4, 220030, г. Минск, Беларусь*
[3)]*Университет гражданской защиты*
Министерства по чрезвычайным ситуациям Республики Беларусь,
ул. Машиностроителей, 25, 220118, г. Минск, Беларусь

Рассмотрена структура компактного узкополосного волоконно-оптического лазера, генерирующего излучение в одной продольной моде (одночастотный лазер). В настоящее время одночастотные лазеры находят широкое применение в оптических системах связи с разделением каналов по длинам волн (технология WDM), оптических датчиках с высоким разрешением, спектрометрах, также они используются для распознавания оптических изображений, детектирования гравитационных волн, создания мощных киловаттных лазерных систем на основе сложения когерентных лазерных пучков и т. д. Предложенный лазер изготовлен на основе короткого отрезка оптического волокна, совместно легированного ионами Er^{3+} и Yb^{3+}. Для создания обратной связи использован резонатор Фабри – Перо,

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Авторы:
Джума Альремеити – младший научный сотрудник[1)], магистрант кафедры лазерной физики и спектроскопии физического факультета[2)]. Научный руководитель – И. А. Гончаренко.
Игорь Андреевич Гончаренко – доктор физико-математических наук, профессор; профессор кафедры естественных наук факультета подготовки научных кадров[3)], профессор кафедры лазерной физики и спектроскопии физического факультета[2)].

Authors:
Juma Alrumeithi, junior researcher[a] and master's degree student at the department of laser physics and spectroscopy, faculty of physics[b].
juma.alrumeithi@tii.ae
https://orcid.org/0000-0002-5716-954X
Igor A. Goncharenko, doctor of science (physics and mathematics), full professor; professor at the department of natural sciences, faculty of postgraduate scientific education[c], and professor at the department of laser physics and spectroscopy, faculty of physics[b].
en@ucp.by
https://orcid.org/0000-0002-8063-8068

сформированный волоконными брэгговскими решетками, которые записаны на концах отрезка легированного волокна. С помощью математического моделирования выполнена оптимизация параметров элементов лазера для достижения генерации на одной продольной моде на длине волны 1550 нм, соответствующей минимальным потерям стандартного телекоммуникационного волокна. Теоретическая модель основана на получении решения для стационарного состояния системы скоростных уравнений для оптических полей с учетом взаимодействия между энергетическими уровнями легированного ионами Er^{3+} – Yb^{3+} кварцевого волокна. Система уравнений решена с использованием итерационного метода в программном коде *Matlab*, что позволило провести анализ заселенности энергетических уровней ионов эрбия и иттербия. Для вычисления функции отражения брэгговских решеток применен разработанный авторами алгоритм на основе метода линий. С использованием программного пакета *Optisystem* произведен расчет динамики установления генерации предложенного лазера на основе легированного ионами Er^{3+} – Yb^{3+} оптического волокна. Показано, что при оптимальных параметрах элементов лазера можно получить одночастотную генерацию в непрерывном режиме с мощностью 0,67 Вт на длине волны 1550 нм при мощности накачки 1 Вт с дифференциальной эффективностью (т. е. эффективностью наклона кривой зависимости мощности сигнала от мощности накачки), равной 67 %.

Ключевые слова: волоконно-оптический лазер; одночастотная генерация; волоконная брэгговская решетка; легированное ионами Er – Yb оптическое волокно; резонатор Фабри – Перо.

A SINGLE-FREQUENCY LASER ON THE BASE OF Er – Yb CO-DOPED OPTICAL FIBER WITH FIBER BRAGG GRATINGS

J. ALRUMEITHI [a, b], ***I. A. GONCHARENKO*** [b, c]

[a] *Technology Innovation Institute, Masdar City 9639, Abu Dhabi, United Arab Emirate*
[b] *Belarusian State University, 4 Niezaliežnasci Avenue, Minsk 220030, Belarus*
[c] *University of Civil Protection, Ministry for Emergency Situations of the Republic of Belarus, 25 Mašynabudaŭnikoŭ Street, Minsk 220118, Belarus*

Corresponding author: J. Alrumeithi (juma.alrumeithi@tii.ae)

The structure of the compact narrow-band fiber lasers operating in a single longitudinal mode (single-frequency lasers) has been considered. The single-frequency fiber lasers are widely used in WDM telecommunication systems, high-resolution sensors, spectrometers, as well as for lightimaging detection, detection of gravitational waves and creation of multikilowatt laser systems based on coherent combining of laser beams. The proposed laser is based on the short length of optical fiber co-doped with Er^{3+} and Yb^{3+} ions. For cavity feedback a Fabry – Perot resonator is formed by using fiber Bragg gratings written on the ends of the doped fiber section. On the base of the numerical simulations the optimisation of the laser elements parameters has been carried out in order to achieve the single longitudinal mode laser generation at 1550 nm wavelength that corresponds to the minimum loss generated by the telecommunication fibers. The theoretical model is based on the steady state solutions of the rate equations resulting from the optical fields and ions interaction inside the Er^{3+} – Yb^{3+} co-doped silica fiber. The mathematical equations are solved using an iterative method by *Matlab* to provide an analysis of the energy levels of erbium and ytterbium ions doping the silica fiber. For the calculation of the fiber Bragg grating reflectivity the original algorithm based on the method of lines has been applied. A series of simulations have been performed by using the *Optisystem* software to obtain the modelling of the dynamic behaviour of the proposed Er^{3+} – Yb^{3+} co-doped fiber laser. The optimised Er^{3+} – Yb^{3+} co-doped fiber laser demonstrated a continuous wave power of 0.67 W at 1550 nm wavelength for a pump power of 1 W with a slope efficiency of 67 %.

Keywords: fiber laser; single-frequency generation; fiber Bragg grating; Er – Yb co-doped fiber; Fabry – Perot resonator.

Introduction

The single-frequency erbium-doped fiber lasers are considered as a key tool due to their high potential applications in WDM telecommunication systems operating with wavelengths around 1.55 μm, high-resolution sensors, spectrometers, for lightimaging detection, detection of gravitational waves and creation of multikilowatt laser systems based on coherent combining of laser beams [1–14]. They are attracting a great deal of interest and high demand because of their ability to provide fiber compatible laser sources with a narrow linewidth (tens of kilohertz), low level of optical noise, high output power, low threshold pump power and a high stability of output parameters. Single-frequency fiber lasers are being actively developed, primarily due to their compactness compared to solid-state counterparts and better output characteristics compared to semiconductor lasers.

Short fiber lengths and wavelength selective filters such as the fiber grating are desired to maintain the stability and provide single-frequency operation to avoid the mode hopping issue. However, a few centimeters long fiber does not provide a satisfactory pump absorption and leads to insufficient and low output power. Although, the expectation of considering high erbium concentrations contributes to increase the pump absorption and provide a high desired gain but the ion-ion interactions leads to the waste of energy by exchanging the energy between the ions of erbium and critically reducing the laser efficiency and deplete the erbium metastable level [15]. In order to provide the solution to these problems the co-doping the gain medium of the fiber with erbium (Er^{3+}) and ytterbium (Yb^{3+}) ions has been proposed [2; 11; 12; 16; 17]. By providing the optimal concentrations of the rare-earth ions, the pair induced energy transfer from Er^{3+} and Yb^{3+} provides the desired pumping requirements for the erbium and allows to rescale the concentrations of the erbium without a high reduction in the efficiency due to the cooperative up conversion. The contribution of Yb^{3+} sufficiently avoids the formation of Er^{3+} clusters and allows an effective contrasts of the energy transfer. Additionally, the Yb^{3+} allows the extension range of the acceptable pump wavelengths and the highest absorption cross section is defined at 980 nm. The pumping of Yb^{3+} at 980 nm provides the best performance and high efficiency to construct a compact fiber lasers.

In this work we propose a theoretical model for the analysis of a compact single-frequency fiber laser based on Er^{3+} – Yb^{3+} co-doped fibers; the continuous wave (CW) operations of single-frequency fiber laser using the Bragg gratings as wavelength filtering components at the ends of the section of the doped fiber and considering the principle of the linear cavity configuration of the Fabry – Perot architecture to generate a high light confinement to satisfy the laser performance requirements. The theoretical work proposed are proved to have a great agreement with the published experimental results associated with compact single-frequency Er^{3+} – Yb^{3+} co-doped fiber lasers.

The main objective of this work is to analyse the possible design of a compact single-frequency high power laser on the base of short length of optical fiber co-doped with Er^{3+} and Yb^{3+} ions. The laser cavity is created by Bragg gratings written on the fiber ends. Short fiber length leads to insufficient pump radiation absorption. The optimal concentrations of these ions should be established by solving propagation-rate equations. Additionally, we provided the numerical analysis of the influence of the fiber Bragg grating parameters on its transmission function. On the base of the analysis the proper selection the parameters will be done to provide the single-frequency regime of the laser generation.

Making the propagation-rate equations for Er – Yb composition

The system of rate equation resulting from the energy level system between the Er^{3+} and Yb^{3+} ions are considered as a complex and difficult to solve without any boundary conditions and estimations. Figure 1 shows a simplified energy level of the system with neglecting some transitions that exhibit the small and less influence on the characteristics of the laser.

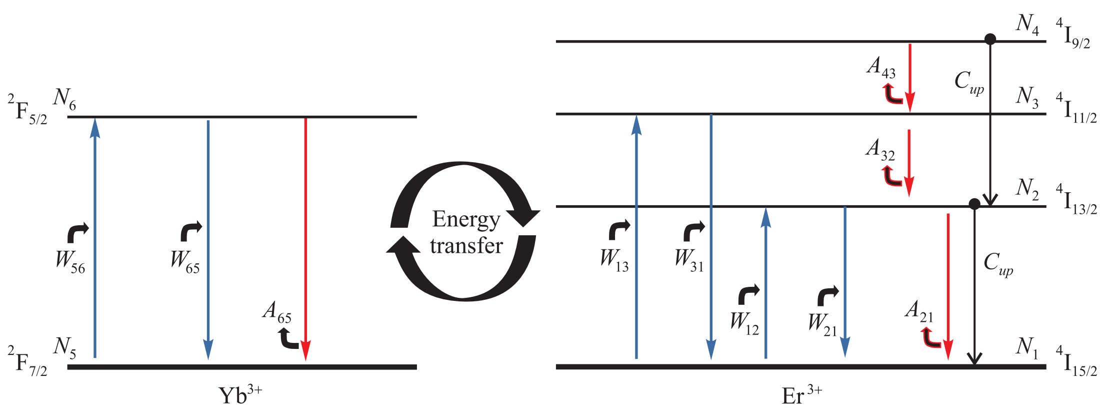

Fig. 1. Simplified energy level diagram of Yb^{3+} and Er^{3+} ions

These assumptions didn't affect significantly the overall results of the laser system performance and characteristics while maintaining the accuracy of the model. As the gain medium is pumped at 980 nm, the system of rate equations resulting can be written as [7; 18; 19]

$$\frac{\partial N_1}{\partial t} = -W_{12}N_1 - W_{13}N_1 + A_{21}N_2 + W_{21}N_2 + C_{up}N_2^2 - C_{cr14}N_1N_4 + C_{up}N_3^2 - C_{cr}N_1N_6,$$

$$\frac{\partial N_2}{\partial t} = W_{12}N_1 - A_{21}N_2 - W_{21}N_2 + A_{32}N_3 - 2C_{up}N_2^2 + 2C_{cr14}N_1N_4,$$

$$\frac{\partial N_3}{\partial t} = W_{13}N_1 - A_{32}N_3 + A_{43}N_4 - 2C_{up}N_3^2 + 2C_{cr14}N_1N_4,$$

$$N_1 + N_2 + N_3 + N_4 = N_{\mathrm{Er}},$$

$$\frac{\partial N_5}{\partial t} = -W_{56}N_5 + A_{65}N_6 + W_{65}N_6 + C_{cr}N_1N_6,$$

$$N_5 + N_6 = N_{\mathrm{Yb}},$$

where N_1, N_2, N_3 and N_4 are the populations of $^4\mathrm{I}_{15/2}$, $^4\mathrm{I}_{13/2}$, $^4\mathrm{I}_{11/2}$ and $^4\mathrm{I}_{9/2}$ energy levels corresponding to the Er^{3+} ions, also N_5 and N_6 are the populations of $^2\mathrm{F}_{7/2}$ and $^2\mathrm{F}_{5/2}$ energy levels corresponding to the Yb^{3+} ions; W_{ij} represents the stimulated transition probabilities (*ij* represents the transition from *i* to *j*); A_{21} and A_{65} terms relats and identifies the spontaneous transition probabilities, also A_{32} and A_{43} represent the non-radiative relaxation probabilities; C_{up}, C_{cr14} and C_{cr} are the transformation of the coefficients associated with the increased frequency and cross-relaxation between the Er^{3+} and Yb^{3+} ions; N_{Er} and N_{Yb} identify and introduce the concentrations of the Er^{3+} and Yb^{3+} ions. It should be noted that for this model we neglected the diverse of mechanisms and conditions that contribute to broaden the energy levels, as well as the associated related temperature instability [7].

Inside of the laser gain medium, the optical field at pump and laser wavelengths propagates in the longitudinal *z*-direction. Therefore, each of these fields will be described by a differential equations governing by the spatial variation of the optical power. The variation of the pump power $P_p\left(z, \nu_p\right)$ and the power of laser radiation $P_{sj}^{\pm}\left(z, \nu_{sj}\right)$ over the length specified for the cavity of the laser is described by the system of differential equations that identified as the propagation equations and can be written as [7; 18; 19]

$$\frac{dP_p\left(z, \nu_p\right)}{dz} = -\Gamma_p\left[\sigma_{\mathrm{Er13}}\bar{N}_1 - \sigma_{\mathrm{Yb56}}\bar{N}_5 - \sigma_{\mathrm{Yb65}}\bar{N}_6\right]P_p\left(z, \nu_p\right) - l_pP_p\left(z, \nu_p\right),$$

$$\frac{dP_{sj}^{\pm}\left(z, \nu_{sj}\right)}{dz} = \pm\Gamma_{sj}\left[\sigma_{\mathrm{Er21}}\left(\nu_j\right)\bar{N}_2 - \sigma_{\mathrm{Er12}}\left(\nu_j\right)\bar{N}_1\right]P_{sj}^{\pm}\left(z, \nu_{sj}\right) \pm$$
$$\pm\, 2h\nu_j\Delta\nu_j\Gamma_{sj}\sigma_{\mathrm{Er21}}\left(\nu_j\right)\bar{N}_2 \pm l_{sj}P_{sj}^{\pm}\left(z, \nu_{sj}\right), \tag{1}$$

$$P_p\left(0, \nu_p\right) = P_{p0},$$

$$P_{sj}\left(0, \nu_j\right) = 0,$$

$$R_2\left(\nu_j\right)P_{sj}^{+}\left(L, \nu_j\right) = P_{sj}^{-}\left(L, \nu_j\right),$$

$$R_1\left(\nu_j\right)P_{sj}^{-}\left(0, \nu_j\right) = P_{sj}^{+}\left(0, \nu_j\right),$$

where $j = 1, \ldots, M$ (*M* represents the number of the longitudinal modes that can be produced by the laser); ν_p, ν_j and l_p, l_{sj} are the frequencies of the radiation and the losses of the pump wave and the generated wave; $R_1\left(\nu_j\right)$ and $R_2\left(\nu_j\right)$ are the reflectance of the cavity mirrors; *L* is the length of the laser cavity; $\sigma_{\mathrm{Er}ij}$ and $\sigma_{\mathrm{Yb}ij}$ are the absorption and emission cross section of the Er^{3+} and Yb^{3+} ions. The sign «plus» in the notation $P_{sj}^{\pm}\left(z, \nu_{sj}\right)$ corresponds to the forward propagation of the optical radiation, respectively the sign «minus» represents the optical radiation propagation in the reverse direction [7]. The overlap integrals between the cross section of the radiation fields and the active region of the waveguided identified as Γ_p and Γ_{sj} based on the assumption of considering the Gaussian intensity distribution of the fields of the pump radiation and the signal radiation can be determined by the following relationship [20]:

$$\Gamma_m = 1 - \exp\left(\frac{a_d^2}{\omega_m^2}\right),$$

where a_d is introduced as the active radius of the fiber, and ω_m is the transverse size of the radiation field, as $m = p$ for the pump wave and $m = sj$ for the corresponding signal wave. The term $2h\nu_j\Delta\nu_j\Gamma_{sj}\sigma_{\mathrm{Er21}}\left(\nu_j\right)\bar{N}_2$ expressed in equation (1) provides the description of the input noise equivalent power [7].

Fiber Bragg grating parameters to provide the single-frequency regime of the laser generation

We start with the numerical analysis of the uniform periodic Bragg grating with the high-reflection (HR). The fiber core radius is R_{co} = 2.88 μm, the refractive index in the core of the first homogenous region is n_{co1} = 1.450 0, and the refractive index of the cladding is n_{cl} = 1.412 5. The number of the grating periods is N = 22 620, the grating period is Λ = 0.537 82 μm, the grating length is L_{FBG} = 24.442 mm, and the relative difference between the grating core refractive indices is Δn_{co} = 0.000 1. The reflectivity of the uniform periodic grating with the selected parameters was assigned in *Matlab* [21; 22] and the simulation result is shown in fig. 2.

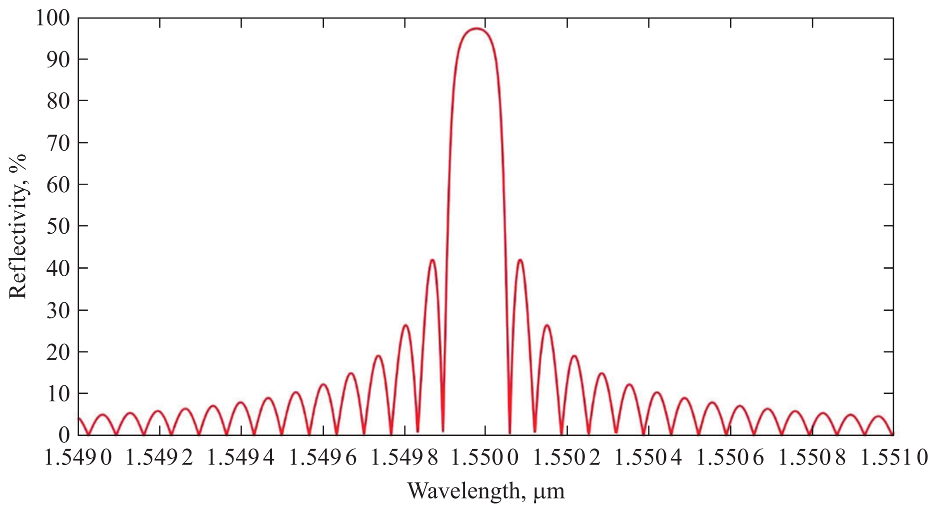

Fig. 2. Reflectivity of a uniform periodic fiber gratings with number of periods N = 22 620 and Δn_{co} = 0.000 1, leading to a total reflection 97 % with narrow central peak wavelength 1550 nm

The modelling results of the uniform periodic grating reflection spectra (Bragg wavelength λ_B = 1550 nm) with a reflectivity of 97 % based on the parameters specified in the simulation, and this considered sufficient for the HR Bragg grating to be implemented and inserted as reflector for the Fabry – Perot cavity feedback.

The second numerical analysis of the uniform periodic grating was performed for the partial reflection (PR) Bragg grating [21; 22]. The fiber core radius is R_{co} = 2.88 μm, the refractive index in the core is n_{co1} = 1.450 0, and the refractive index of the cladding is n_{cl} = 1.412 5. The number of the grating periods is N = 6610, the grating period is Λ = 0.537 82 μm, the grating length is L_{FBG} = 7.109 98 nm, and the relative difference between the core refractive indices is Δn_{co} = 0.000 1. The modelling results of the uniform periodic grating reflection spectra (Bragg wavelength λ_B = 1550 nm) with a reflectivity of 55 % based on the parameters specified in the simulation is shown in fig. 3.

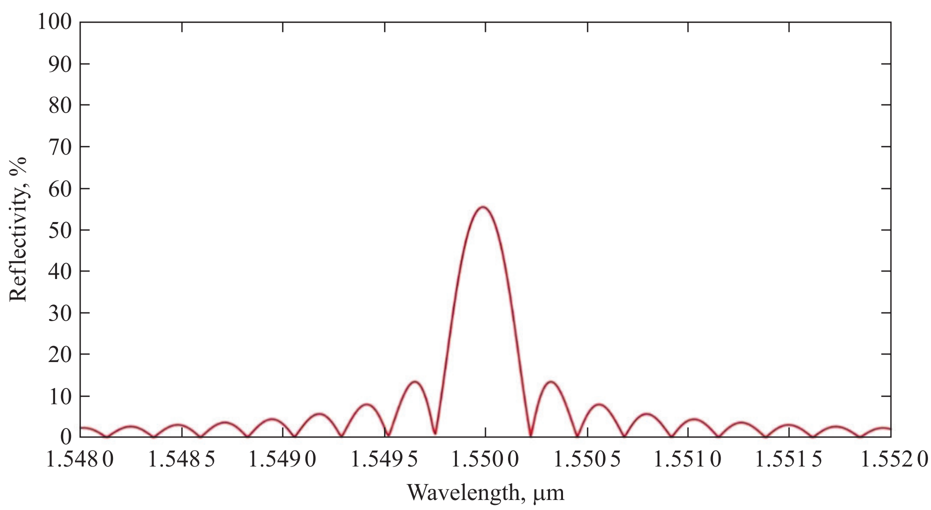

Fig. 3. Reflectivity of a uniform periodic fiber gratings with number of periods N = 6610 and Δn_{co} = 0.000 1, leading to a total reflection 55 % with narrow central peak wavelength 1550 nm

Therefore, the results considered are reliable and convenient for the PR Bragg grating and could be implemented and inserted as the partial reflector of the Fabry – Perot cavity for the outcoupling.

Following to the calculations for the Fabry – Perot cavity, the transmittance maxima corresponding to the Fabry – Perot resonance is observed to overlap with desired reflectivity spectrum based on the physical length of the cavity [23]. Figure 4 shows the fiber cavity transmittance spectrum using the uniform Bragg grating, the length of the Fabry – Perot cavity $L = 1$ cm, and neglecting the absorption associated with the Bragg grating as reflectors for the proposed configuration. The peak wavelength of the uniform fiber Bragg grating is centered at 1550 nm. The cavity transmission spectra (see fig. 4) were obtained based on the proposed configuration of the Fabry – Perot cavity in order to produce the single mode fiber laser generation. The central resonance of the Fabry – Perot cavity is at the desired Bragg wavelength (λ_B = 1550 nm) of the uniform gratings.

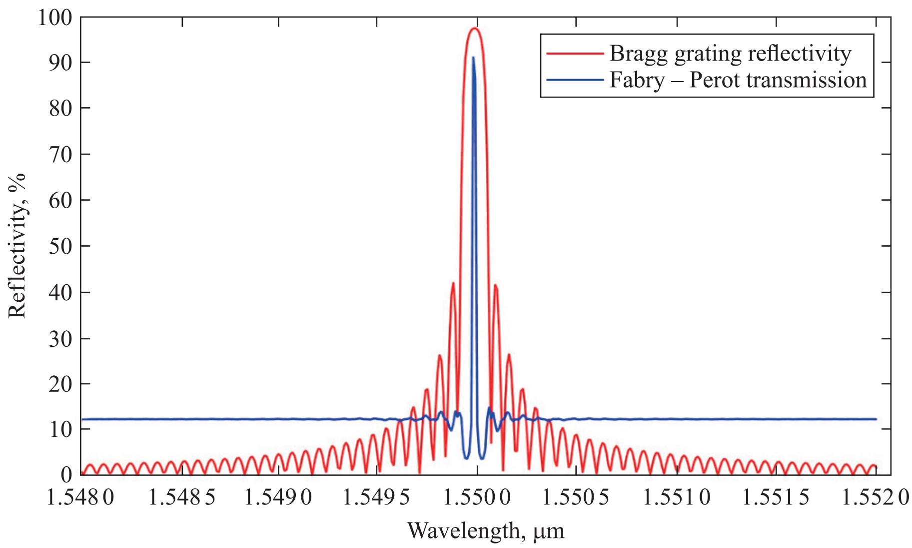

Fig. 4. Transmittance spectra of the Fabry – Perot fiber cavity formed by uniform fiber Bragg gratings

Fiber laser setup

The schematic digram of the proposed Er^{3+} – Yb^{3+} co-doped fiber laser is shown in fig. 5. The pump source is the laser diode operating at 980 nm, the beam of this fiber laser system is collimated first and then propagated through the optical isolator to secure the system from any back reflections from the fiber ends that can damage the input source identified as the diode laser for the proposed fiber laser optical system. The pump radiation is focused directly into the Er – Yb fiber through the first HR fiber Bragg grating, which is 97 % reflecting at 1550 nm. The section of the 1 cm length Er – Yb fiber with a numerical aperture of 0.32 and the outcoupling of the light is based on the output PR fiber Bragg grating, which has 55 % reflecting at 1550 nm.

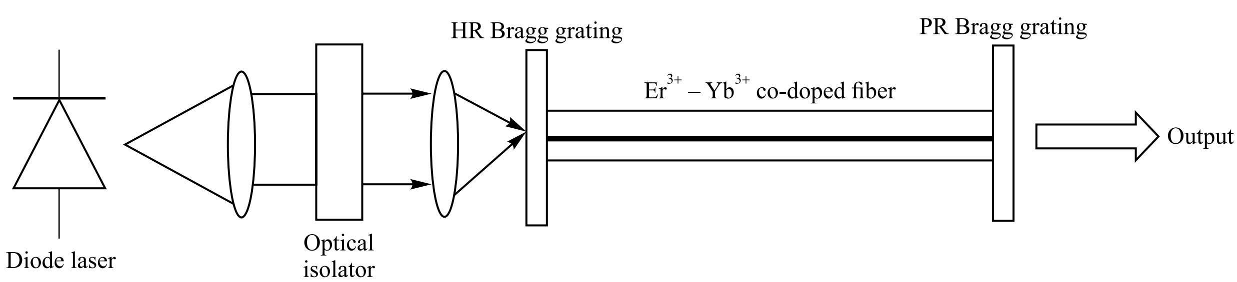

Fig. 5. Schematic diagram of the Er^{3+} – Yb^{3+} co-doped fiber laser

Fiber laser simulation

The proposed fiber laser setup is modelling by using the *Optisystem* software to reach the suitable design that should be implemented. *Optisystem* software allows to establish the reliable optical design to determine the fiber laser architecture for the Er^{3+} – Yb^{3+} co-doped fiber laser. Evaluate cost and performance by calculating the output power, maximum noise figure, maximum gain ripple and minimum pump power depend on the optical system specifications such as pump wavelength range, passive component losses, component costs and so on. The results of the simulation is shown in fig. 6. Er^{3+} – Yb^{3+} fiber laser is excited and pumped by the laser diode at wavelength of 980 nm. The power of the pump source was swept from 0 to 1 W by the CW diode laser. The pump radiation was focused into the Er^{3+} – Yb^{3+} co-doped fiber through the HR fiber Bragg grating. This component is introduced as the bidirectional reflective filter and can be considered as the fiber Bragg grating for the *Optisystem* platform. The HR fiber Bragg grating has the 97 % reflecting at 1550 nm and 99 % transmitting at 980 nm. Two optical delays were introduced in the setup of the optical system to generate optical signal delay in order to enable the calculations of the simulation. A length of 0.01 m of Er^{3+} – Yb^{3+} fiber is introduced with a numerical aperture of 0.32, Er^{3+} ions density in $5.0 \cdot 10^{25}$ ions per 1 m^3 and Yb^{3+} ions density in $6.25 \cdot 10^{26}$ ions per 1 m^3 of the Er^{3+} – Yb^{3+} co-doped fiber optical components simulates the fiber laser setup when solving the numerically the rate and propagation equations for a steady-state case and considering the non-linear phase changes by propagating the signal by using the non-linear Schrödinger equation. The outcoupling of the fiber laser is based on the PR fiber Bragg grating (55 % reflecting at 1550 nm). The buffer selector is used for selecting the signal data associated with a specified iteration in a series of iterations based on the proposed fiber laser setup. Convert to sampled signals defines the input signal type to be converted to sampled signals.

Analysis of the simulation results

The CW power of 0.67 W at 1550 nm is demonstrated for a launched power of 1 W with a slope efficiency of 67 %. Figure 6 shows the optimised design of the optical system for the compact Er^{3+} – Yb^{3+} co-doped fiber laser and the results indicated for the best slope efficiency with the length of 0.01 m. The performance of the proposed setup will start to degrade as we increase the length of the fiber. Figure 7 shows the results of the obtained output power. The noise power at the output of the optical system by using the built in *Optisystem* optical power meter is presented in fig. 8.

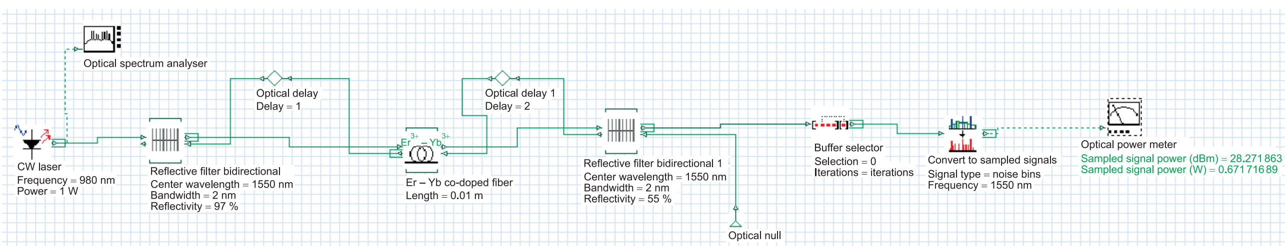

Fig. 6. Optisystem simulation setup for Er^{3+} – Yb^{3+} co-doped fiber laser with the optimal length $L = 0.01$ m

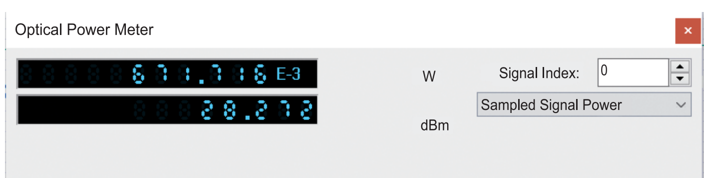

Fig. 7. Output power measured by the built in optical power meter (*Optisystem* software)

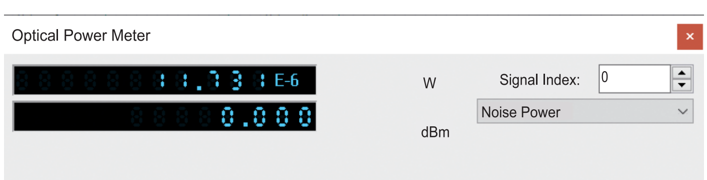

Fig. 8. Noise power at the output measured by the built in optical power meter (*Optisystem* software)

Evaluation of Er^{3+} – Yb^{3+} co-doped fiber laser

The demonstrated Er^{3+} – Yb^{3+} co-doped fiber laser for single-frequency operation can be determined as the function of grating reflectivity and resonator length. In all calculations discussed in this research, we assumed the fiber laser having pair of Bragg gratings without loss, the laser wavelength of 1550 nm based on the uniform Bragg grating reflectivity and the fiber with the core radius R_{co} = 2.88 μm, core refractive index of n_{co} = 1.4500 and cladding refractive index of n_{cl} = 1.4125. The cavity length is L = 1 cm. We have considered the strong pump regime, which results in high lasing power based on the optical system components and properties. The reflectivity of the Bragg grating reflectors written in a single-mode fiber laser is selected to provide the single mode operation. The uniform Bragg reflectors are modelled using the method of lines theory. Uniform Bragg gratings reflectivity of 97 % is demonstrated for the HR fiber Bragg grating and 55 % for the PR fiber Bragg grating to couple out the light from the end of the fiber laser system. The cavity configuration for the single-mode operation based on the Fabry – Perot resonator demonstrated a dominant lasing mode at the central resonance (Bragg wavelength λ_B = 1550 nm). Analysis of the reflection of uniform Bragg gratings is treated according to the method of lines theory approach. The proposed configuration of the uniform Bragg reflectors results in the single-frequency operation of the 1 cm section of the Er^{3+} – Yb^{3+} co-doped fiber laser. A length of 0.01 m of Er^{3+} – Yb^{3+} fiber is established with the numerical aperture of 0.32, Er^{3+} ion density is $5.0 \cdot 10^{25}$ ions per 1 m^3 and Yb^{3+} ions density is $6.25 \cdot 10^{26}$ ions per 1 m^3. A CW power of 0.67 W at 1550 nm is demonstrated successfully for a launched power of 1 W with a slope efficiency of 67 %. The laser slope efficiency is found to be maximum at the proposed compact Er^{3+} – Yb^{3+}co-doped fiber laser based on the Bragg gratings. This configuration and design were chosen to achieve the high slope efficiency and reasonable stability with respect to the desired frequency at 1550 nm. Er^{3+} – Yb^{3+} co-doped fiber laser was designed and tested using the analysis of the *Optisystem* software to demonstrate reliable numerical results. All the modeled components and parameters for the demonstrated Er^{3+} – Yb^{3+} co-doped fiber laser are shown in the table.

Parameters of Er^{3+} – Yb^{3+} co-doped fiber laser

Parameter	Symbol	Value
Pump wavelength	λ_p	980 nm
Signal wavelength	λ_s	1550 nm
Cavity length	L	1 cm
Pump power	P_p	1 W
Core radius	R_{co}	2.88 μm
Doping radius	R_D	2.88 μm
Core refractive index	n_{co}	1.4500
Cladding refractive index	n_{cl}	1.4125
Numerical aperture	NA	0.32
HR fiber Bragg grating reflectivity	FBG_{HR}	97 %
PR fiber Bragg grating reflectivity	FBG_{PR}	55 %
HR fiber Bragg grating length	FBG_{HR_L}	24.44 mm
PR fiber Bragg grating length	FBG_{PR_L}	7.11 mm
Fiber laser total length	$FLaser_L$	4.155 cm
Temperature	T	300 K
Bandwidth	B_w	2 nm
Emission lifetime of Er^{3+}	Er_τ	10 ms
Emission lifetime of Yb^{3+}	Yb_τ	1.5 ms
Energy transfer coefficient	K_{1242}	$5.283 \cdot 10^{24}$ m^3/s
	K_{3134}	$5.283 \cdot 10^{24}$ m^3/s
	K_{1365}	$0.344 \cdot 10^{24}$ m^3/s

Ending table

Parameter	Symbol	Value
Non-radiative transitions	Anr_{43}	$1 \cdot 10^9$ s^{-1}
	Anr_{32}	$1 \cdot 10^9$ s^{-1}
Total population of Er^{3+}	N_{Er}	$5.0 \cdot 10^{25}$ ions per 1 m^3
Total population of Yb^{3+}	N_{Yb}	$6.25 \cdot 10^{26}$ ions per 1 m^3
Sampled signal power	S_p	0.67 W
Noise threshold	N_T	–100 dB
Noise dynamic	N_D	3 dB
Total power losses	S_L	0.33 W
Efficiency	$FLaser_\eta$	67 %

Conclusions

A theoretical model of the Er^{3+} – Yb^{3+} co-doped fiber laser device was established and developed. The model was based on the steady state solutions of the rate equations resulting from the optical fields and ions interaction inside the Er^{3+} – Yb^{3+} co-doped silica fiber. Initially the steady state rate equations were solved by *Matlab* program based on the method of rate equation pair solver. The numerical analysis of the fiber Bragg gratings configuration to establish a single-frequency fiber laser were computed by applying the method of lines. The optical resonator considered as a Fabry – Perot resonator configuration bases on the Er^{3+} – Yb^{3+} doped silica fiber to obtain a single longitudinal mode. The simulations and modelling results of the Fabry – Perot transmission spectrum of the uniform periodic Bragg grating and apodised fiber grating were obtained with *Matlab* software. The optimised Er^{3+} – Yb^{3+} co-doped fiber laser demonstrated the CW generation with the power of 0.67 W at 1550 nm for a launched power of 1 W with a slope efficiency of 67 %.

The theoretical model presented and developed in this research neglects the Bragg gratings reflector losses. However, as expected there is a very small slight difference between the proposed results and experimental data discussed in this work. The difference is due to the computing different software and programs that consider different algorithms and parameters associated with the optical system components of executing and evaluating the performance of Er^{3+} – Yb^{3+} co-doped fiber laser.

In conclusion a sufficient solid theoretical model of Er^{3+} – Yb^{3+} co-doped fiber laser was successfully established and developed. The performance of the laser in terms of the obtained sampled power demonstrated an increase of approximately 16 % efficiency over the previously published and discussed experimental data presented in this research work.

References

1. Kringlebotn JT, Morkel PR, Reekie L, Archambault J-L, Payne DN. Efficient diode-pumped single-frequency erbium : ytterbium fiber laser. *IEEE Photonics Technology Letters*. 1993;5(10):1162–1164. DOI: 10.1109/68.248414.

2. Di Pasquale F. Modeling of highly-efficient grating-feedback and Fabry – Perot Er^{3+} – Yb^{3+} co-doped fiber lasers. *IEEE Journal of Quantum Electronics*. 1996;32(2):326–332. DOI: 10.1109/3.481880.

3. Bonfrate G, Vaninetti F, Negrisolo F. Single-frequency MOPA Er^{3+} DBR fiber laser for WDM digital telecommunication systems. *IEEE Photonics Technology Letters*. 1998;10(8):1109–1111. DOI: 10.1109/68.701518.

4. Yamashita S, Hsu K. Single-frequency, single-polarization operation of tunable miniature erbium : ytterbium fiber Fabry – Perot lasers by use of self-injection locking. *Optics Letters*. 1998;23(15):1200–1202. DOI: 10.1364/OL.23.001200.

5. Nielsen TG, Hodel W, Weber HP, Iocco A, Costantini DM, Limberger HG, et al. Stable and widely tunable all-fiber Pr^{3+}-doped CW laser system using fiber Bragg gratings. *Optics Letters*. 1999;24(9):614–616. DOI: 10.1364/OL.24.000614.

6. Pilipovich VA, Esman AK, Goncharenko IA, Kuleshov VK. High-speed continuous tuneable fibre and waveguide lasers with controllable Bragg grating. *Optics Communications*. 2002;203(3–6):289–294. DOI: 10.1016/S0030-4018(01)01597-8.

7. Pilipovich VA, Esman AK, Goncharenko IA, Kuleshov VK. Lasing dynamics of tunable single-frequency fiber-optic and waveguide lasers. *Journal of Optical Technology*. 2003;70(3):173–176. DOI: 10.1364/JOT.70.000173.

8. Abbott BP, Abbott R, Adhikari R, Ajith P, Allen B, Allen G, et al. LIGO: the laser interferometer gravitational-wave observatory. *Reports on Progress in Physics*. 2009;72(7):076901. DOI: 10.1088/0034-4885/72/7/076901.

9. Ma Yanxing, Wang Xiaolin, Leng Jinyong, Xiao Hu, Dong Xiaolin, Zhu Jiajian, et al. Coherent beam combination of 1.08 kW fiber amplifier array using single frequency dithering technique. *Optics Letters*. 2011;36(6):951–953. DOI: 10.1364/OL.36.000951.

10. Bai Xiaolei, Sheng Quan, Zhang Haiwei, Fu Shijie, Shi Wei, Yao Jianquan. High-power all-fiber single-frequency erbium – ytterbium co-doped fiber master oscillator power amplifier. *IEEE Photonics Journal*. 2015;7(6):7103106. DOI: 10.1109/JPHOT.2015.2490484.

11. Fu Shijie, Shi Wei, Feng Yan, Zhang Lei, Yang Zhongmin, Xu Shanhui, et al. Review of recent progress on single-frequency fiber lasers [Invited]. *Journal of the Optical Society of America B*. 2017;34(3):A49 – A62. DOI: 10.1364/JOSAB.34.000A49.

12. Sompo JM, Grobler M, Kaboko JJM. Influence of upconversion and excited state absorption on the performance of an erbium-ytterbium doped DFB fiber laser. In: *2019 IEEE Africon; 2019 September 25–27; Accra, Ghana*. [S. l.]: IEEE; 2019. p. 1–5. DOI: 10.1109/AFRICON46755.2019.9134018.

13. Koptev MYu, Egorova ON, Medvedkov OI, Semjonov SL, Galagan BI, Sverchkov SE, et al. Narrow-linewidth single-frequency ytterbium laser based on a new composite Yb^{3+}-doped fiber. *Photonics*. 2022;9(10):760. DOI: 10.3390/photonics9100760.

14. Tao Yue, Jiang Man, Liu Liu, Li Can, Zhou Pu, Jiang Zongfu. More than 20 W, high signal-to-noise ratio single-frequency all-polarization-maintaining hybrid brillouin/ytterbium fiber laser. *Journal of Lightwave Technology*. 2023;41(2):678–683. DOI: 10.1109/JLT.2022.3215655.

15. Wagener JL, Wysocki PF, Digonnet MJF, Shaw HJ, Di Giovanni DJ. Effects of concentration and clusters in erbium-doped fiber lasers. *Optics Letters*. 1993;18(23):2014–2016. DOI: 10.1364/OL.18.002014.

16. Karasek M. Optimum design of Er^{3+} – Yb^{3+} co-doped fibers for large-signal high-pump-power applications. *IEEE Journal of Quantum Electronics*. 1997;33(10):1699–1705. DOI: 10.1109/3.631268.

17. Kringlebotn JT, Archambault J-L, Reekie L, Townsend JE, Vienne GG, Payne DN. Highly-efficient, low-noise grating-feedback Er^{3+} : Yb^{3+} co-doped fibre laser. *Electronics Letters*. 1994;30(12):972–973. DOI: 10.1049/el:19940628.

18. Hsu K, Miller CM, Kringlebotn JT, Taylor EM, Townsend J, Payne DN. Single-mode tunable erbium : ytterbium fiber Fabry – Perot microlaser. *Optics Letters*. 1994;19(12):886–888. DOI: 10.1364/OL.19.000886.

19. Cucinotta A, Selleri S, Vincetti L, Zoboli M. Numerical and experimental analysis of erbium-doped fiber linear cavity lasers. *Optics Communications*. 1998;156(4–6):264–270. DOI: 10.1016/S0030-4018(98)00432-5.

20. Desurvire E, Simpson JR. Amplification of spontaneous emission in erbium-doped single-mode fibers. *Journal of Lightwave Technology*. 1989;7(5):835–845. DOI: 10.1109/50.19124.

21. Goncharenko IA, Helfert SF, Pregla R. General analysis of fibre grating structures. *Journal of Optics A: Pure and Applied Optics*. 1999;1(1):25–31. DOI: 10.1088/1464-4258/1/1/003.

22. Goncharenko IA, Helfert SF, Pregla R. Analysis of nonlinear properties of fibre grating structures. *AEÜ – International Journal of Electronics and Communications*. 1999;53(1):25–31.

23. Born M, Wolf E. *Principles of optics: electromagnetic theory of propagation interference and diffraction of light*. 6^{th} edition. Oxford: Pergamon Press; 1980. Chapter 7, Elements of the theory of interference and interferometers; p. 256–260.

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