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МОДЕЛИРОВАНИЕ ТЕПЛОВЫХ ЭФФЕКТОВ НА ПОЛЯРИЗАЦИОННЫЕ ПЕРЕКЛЮЧЕНИЯ В ПОВЕРХНОСТНО ИЗЛУЧАЮЩИХ ПОЛУПРОВОДНИКОВЫХ ЛАЗЕРАХ

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Аннотация. Развита феноменологическая модель, описывающая наведенную анизотропию коэффициента усиления поверхностно излучающих полупроводниковых лазеров (VCSEL) для режима поперечной одномодовой генерации в виде полинома второго порядка по степеням плотности инжекционного тока, где коэффициенты разложения являются неявными функциями температуры. Модель основана на анализе данных теоретических и экспериментальных исследований динамики тепловых процессов в VCSEL. Из общего вида зависимостей следует, что в одномодовом режиме работы VCSEL может иметь не более двух точек поляризационного переключения. Работоспособность модели продемонстрирована на базе качественного анализа опубликованных ранее серий экспериментальных данных по температурным зависимостям положения точек поляризационного переключения. Для коротковолновых излучателей такие зависимости являются монотонными и без особых проблем описываются за счет относительного смещения кривых, определяющих анизотропию коэффициентов усиления для ортогонально-поляризованных мод. Для длинноволновых VCSEL, данные по которым опубликованы в литературе, ситуация оказывается гораздо более сложной: изменяется не только относительное расположение кривых, но и их «кривизна», связанная с квадратичным членом. При этом оказалось возможным объяснить практически неизменное положение одной из точек поляризационного переключения.

Ключевые слова: поляризационное переключение; поверхностно излучающий полупроводниковый лазер; VCSEL; анизотропия; температурная зависимость точек поляризационного переключения.

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MODELLING THERMAL EFFECTS ON POLARISATION SWITCHING IN SURFACE-EMITTING SEMICONDUCTOR LASERS

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Abstract. A phenomenological model has been developed that describes the induced anisotropy of the vertical-cavity surface-emitting laser (VCSEL) gain for the single-mode mode in the form of a second-order polynomial with respect to the degrees of density of the injection current, where the decomposition coefficients are implicit functions of temperature. The model is based on the analysis of data from theoretical and experimental studies of the dynamics of thermal processes in VCSEL. From the general view of the dependencies, it follows that in single-mode operation, a VCSEL cannot have more than two polarisation switching points, and of different types. The performance of the model is demonstrated on the basis of a qualitative analysis of previously published series of experimental data on temperature dependencies of the position of polarisation switching points. For short-wave emitters, such dependencies are monotonic and can be easily described by the relative shift of the curves that determine the anisotropy of the gain for orthogonally polarised modes. For long-wavelength VCSELs, the data for which have been published in the literature, the situation turns out to be much more complicated: not only the relative location of the curves changes, but also their «curvature», which is mainly associated with the quadratic term. At the same time, it turned out to be possible to explain the almost constant position of one of the points of polarisation switching.

Keywords: polarisation switching; VCSEL; anisotropy; temperature dependence of polarisation switching points.

Introduction

In the works [1–4] it was consistently demonstrated that all the basic regularities of polarisation switching (PS) in vertical-cavity surface-emitting lasers (VCSELs) can be described within the framework of the approach when the formation of radiation in the laser cavity is considered as a sequential amplification of partially polarised radiation, taking into account the anisotropy of gain and (or) losses [5]. With this approach, the PS acquires the character of a fully deterministic transition from a single linear polarisation to an orthogonal polarisation through a chain of partially polarised states with a sequential change in the injection current near the value for which the induced anisotropy of gain and (or) losses turns to zero. This mechanism is fundamentally different from the interpretation of PS within the framework of the polarisation mode method, when PS is considered as the result of bistable competition of two independent linearly polarisation modes [6; 7], but provides a relatively simple and physically sufficiently transparent interpretation of the phenomena accompanying PS [1–4].

Perhaps the most significant limitation of the applicability of the results of [1–4] was the phenomenological approximation of the linear dependence of the anisotropy of gain and (or) losses for a separate polarisation component [1; 3] on the density of the injection current, although the results of calculations [1; 5] show that the PS is observed in areas where the relative anisotropy becomes noticeably less than 10^{-3}, i. e. the linear approximation in the vicinity of the point of PS is quite correct at least from the point the vision of mathematics. However, this does not mean that the linear dependence is the only possible one in the global sense, i. e. for the entire range of changes in the density of the injection current.

In particular, the linear dependence should result in the presence of no more than one PS point for the single-mode mode, while in reality two points can be observed, and with different types of PSs [8; 9]. At the same time, according to the authors [10], for some emitters, the gain anisotropy is close to linear only in the region of exceeding the threshold by 50 %, but its relative value can be so small ($\sim 3 \cdot 10^{-5}$–$5 \cdot 10^{-5}$) that both polarisation modes have a noticeable output power.

Thus, the linear approximation is quite reasonable in the field of PS points, but as part of the further development of our approach, the question arises about a more general dependence of the anisotropy of gain and (or) losses on the density of injection current, which is based on the analysis of physical processes occurring in the region of PS points. The results of this analysis of numerous studies of polarisation dependencies in VCSELs suggest [11; 12] that anisotropy of gain and (or) losses can be a consequence of three main groups of phenomena:

- technological, related to the processes of growing multilayer semiconductor structures, which lead to the occurrence of stresses and, as a result, the appearance of induced anisotropy [13; 14];
- electro-optical (and other related processes) determined by the action of sufficiently strong electric fields generated by displacement voltage and pump currents, and the inhomogeneity of the distribution of injection current into the cross-section of the laser beam being formed [15; 16];

• thermal, which can also be divided into two groups: stationary, determined by the global (average) temperature of the laser diode, which is tried to be kept constant in the temperature control mode, and which acts as a parameter of the emitter, and dynamic, associated with the conversion of part of the dynamic pumping energy into thermal energy [17; 18], since the dependence of the output power on the current in most cases is obtained by a monotonous change of the latter within the specified limits for times 10^{-3}–10^{-6} s.

It should be noted that the authors of this publication have not been able to find any data in the literature that would directly link the range of these phenomena with the anisotropy of the properties of semiconductor emitters. Therefore, we will use the already proven method [19] of phenomenological modelling, substantiating the option of choosing the dependence of the anisotropy of gain and (or) losses on the density of the injection current based on the analysis of the above sources of induced anisotropy in VCSELs. And then we use the resulting dependency to analyse the available experimental data.

Phenomenological temperature dependence of induced gain anisotropy

It is not necessary to analyse the technological sources of anisotropy of the active medium, they are very diverse [20; 21], we will only note that due to the high Q-factor of the resonator VCSELs, even relatively weak anisotropy of gain and (or) losses leads to the fact that even in the presence of axial symmetry of the laser system itself, the output is, as a rule, linearly polarised radiation. At the same time, it should be emphasised that this «basic» level of anisotropy may depend on temperature [20; 21], but under temperature control conditions the temperature of the emitter is kept constant and this part of the anisotropy can be considered as constant for a given temperature and independent of the density of the injection current.

Electro-optical effects are a consequence of stresses in the active medium and are described by microscopic models [22], the results are usually numerical, but the general conclusion is that the induced anisotropy is relatively small, the value of which can be estimated, for example, by the difference in the spectra of the gain coefficients for orthogonal polarisation modes [22]. Therefore, the influence of stresses is usually taken into account phenomenologically, as is done, for example, in the framework of the most popular SFM (spin – flip model) approach [6] by introducing coefficients describing dichroism and birefringence. From this point of view, the introduction of a linear function for induced anisotropy in [1] is quite in line with existing approaches, but even within the framework of the «advanced» SFM model [23], the possibility of introducing nonlinear components of dichroism and birefringence is considered. As we shall see, this problem loses some relevance after considering the issues related to thermal processes.

In general, thermal processes are extremely relevant in describing the functioning of semiconductor laser emitters [20; 21]. In particular, a strong increase in the injection current leads to a strong heating of the laser diode and, as a result, a drop in the lasing efficiency up to a complete failure of the lasing (thermal rollover [24]). However, it is the thermal mechanism associated with the different magnitude of the «red» shift of the gain spectra and lasing modes that was originally proposed [17; 25] to explain polarisation switches. Although this mechanism has not been able to explain all the features of PSs, the interest in studying the influence of thermal processes on the output radiation characteristics of VCSELs continues unabated.

However, the analysis of the results obtained in this area should be treated with some caution for two main reasons. Simulations of the simultaneous effect of temperature on the electronic properties of the VCSEL and heat transfer processes with significantly different dynamic characteristics lead to very complex models, so certain (sometimes significant) simplifications are used in practice, which can relate to both the processes of heat transfer and its effect on the electronic properties of the emitters. In other words, if the main attention is paid to the adequacy of the temperature distribution inside the semiconductor emitter, the description of the process of laser radiation formation is simplified, and vice versa. The only thing that unites the vast majority of works is the use of various variants of the SFM model [18; 26–28], but this is exactly what creates certain inconveniences, since this work uses a different approach to the mechanism of polarised radiation formation and some of the specific conclusions simply cannot be used directly. Therefore, in order to further substantiate the suitability of our phenomenological model of induced anisotropy, we will mainly use the results and conclusions of the three basic works.

The first two contain the results of detailed experimental studies of the effect of temperature on polarisation switches in short-wave [8] and long-wave [9] VCSELs, which have not been appropriately interpreted by the authors (given as a fact) and will be used in the future to analyse the suitability of the proposed model. And the third work [29] includes not only the results of modelling thermal effects (again on the basis of the SFM model), but also a comparison of the obtained results with the results of other authors in terms of their generality, so that the conclusions of the authors [29] are, in a certain sense, universal. Let's focus on the most important ones for our study.

1. There are two main sources of heat:

• linear in terms of injection current density, associated both with non-optical transitions in the active medium and with contact currents (such as the Peltier effect) in a multilayer system;

• quadratic in terms of injection current density associated with the Joule – Lenz effect.

2. As the temperature changes, the PS points shift in the same way as the threshold current, but this is not necessarily in the same way, since the PS points may disappear from the specified range altogether.

3. In the transverse single-mode lasing mode, no more than two PS points can be observed.

4. At a relatively small excess of the lasing threshold (depending on the type of emitter), the linear component of the contribution predominates, but with an increase in the injection current, the influence of the Joule – Lenz effect (quadratic term) increases.

Let's take a closer look at the effect of the Joule – Lenz effect. Apparently, the authors [30] were among the first to pay attention to the influence of this effect when studying the energy and spectral characteristics of a short-wave quantum-sized emitter in the low temperature region. However, this effect is very inertial and should manifest itself relatively weakly in dynamics, while the practice of obtaining the dependence of polarisation characteristics on the value of the injection current is usually associated with measurements with a sequential linear change in the magnitude of the current. It should be noted that when analysing the published experimental data, it is difficult to find references to the rate of change in the value of the injection current, but the results of numerical simulation [29] show that the approximation of quasi-stationary excitation is well performed up to the rates of rise of the injection current of the order of 1.0–1.5 mA/μs. Obviously, it is precisely the dynamic nature of obtaining polarisation dependencies that is associated with the conclusion of the work on heating modelling that the main contribution is made by linear processes, and the contribution of quadratic processes becomes noticeable only at high injection currents [24; 29].

Though, when using real stationary injection processes, the PS time, according to the authors [31], ranged from a few to more than a thousand seconds from the moment the current was switched on (the current rise time was 15 ms [31]). Such an effect can be attributed to the actual manifestation of warming up due to the Joule – Lenz effect.

On the basis of the above, a second-order polynomial can be chosen as the initial dependence of the induced anisotropy on the density of the injection current, but the coefficients of such a polynomial should be considered as a function of the temperature of the emitter. It should be recalled that thermal effects have always played a significant role in the formation of polarisation effects in VCSELs. For example, thermal effects were associated with the appearance of a second PS point [32] for the same transverse lasing mode. Therefore, the study of the conditions of manifestation of two PS points for the single-mode mode can be correlated with the influence of thermal processes of different nature on the polarisation properties of the output radiation of VCSELs.

However, speaking about the temperature dependence of the polynomial decomposition coefficients of induced anisotropy, it is necessary to agree on what kind of temperature we are talking about. It's necessary to point, theoretical calculations [18; 24; 28; 29] show a significant inhomogeneity in the temperature distribution inside the emitter, and attempts to relate the effective temperature of the PS to the temperature dependence of the threshold current are not very consistent, as will be demonstrated below, with the available experimental data. Moreover, indirect measurements of the temperature of the active layer based on shifts in the electroluminescence spectra [33] indicate an increase in the inhomogeneity of the temperature distribution with an increase in the output power of generation. Therefore, within the framework of the phenomenological model, a certain effective value can act as a temperature, which can be taken as the temperature of the substrate, especially since it is this value that is controlled during temperature control.

Thus, the main task of this work can be formulated as follows: within the framework of the previously developed approach [1–4], the system of equations formulated in [1; 3] will be used to describe the polarisation properties of the output radiation of VCSELs, and to simplify the calculations, the entire orientation anisotropy can be transferred [2] to the gain of a separate polarisation component, which can be presented in a general form [1]:

$$G(\psi) = g_0 (N - N_{\text{tr}}) \left(1 + k_x \cos^2 \psi + k_y \sin^2 \psi\right), \tag{1}$$

where the first term of the expression defines the isotropic gain, and k_x and k_y define the anisotropic corrections to the gain, with the x and y axes lying in the plane of the wavefront and may correspond to the polarisation directions of the *TE* and *TM* modes, although, as we shall see, this is by no means a prerequisite. In accordance with the assumptions made above, let us

$$k_{x(y)} = k_{0x(0y)} + jk_{1x(1y)} + j^2 k_{2x(2y)}. \tag{2}$$

Now the main problem comes down to the determination of k_{mn} coefficients, or at least their relations, which is more characteristic of phenomenological models. To simplify the further analysis, we convert the expression (1) to the form

$$G(\psi) = g_0(N - N_{\text{tr}})(1 + K_1 + K_2 \cos 2\psi), \tag{3}$$

where $K_1 = \frac{1}{2}\left[k_{0x} + k_{0y} + j\left(k_{1x} + k_{1y}\right) + j^2\left(k_{2x} + k_{2y}\right)\right]$ and $K_2 = \frac{1}{2}\left[k_{0x} - k_{0y} + j\left(k_{1x} - k_{1y}\right) + j^2\left(k_{2x} - k_{2y}\right)\right]$.

Here, the first term K_1 describes the isotropic contribution of the induced amplification, and K_2 describes its anisotropy. It should be noted right away that for real emitters there is a close to linear dependence of the output radiation power on the injection current density in a fairly wide range of change in the injection current density j, although for long-wave VCSELs it can be assumed that this range will be somewhat smaller (compare the results [8] and [9]), which is associated with a stronger influence of thermal effects in long-wave semiconductor emitters [21]. However, a sufficiently wide range of linearity of the output characteristics of VCSELs makes it possible to formulate the following assumption:

$$\left|k_{0x} + k_{0y}\right| \gg \left|k_{1x} + k_{1y}\right| \gg \left|k_{2x} + k_{2y}\right|,$$

which means the above-mentioned consequence of modelling thermal processes of increasing the effect of the Joule – Lenz effect only in the region of large injection currents [29]. Moreover, according to the analysis of experimental data for emitters generating at a wavelength of 850 nm [34], the main contribution to the heating of emitters when the injection current is passed is made by linear processes in a wide range of exceeding threshold values and a wide range of temperatures. Therefore, if we take as a basis the estimates and the results of the simulation of the work [1], then the following ratio in relative units can be taken as an initial estimate:

$$\frac{k_{1x} + k_{1y}}{k_{0x} + k_{0y}} \sim 10^{-1}, \ \frac{k_{2x} + k_{2y}}{k_{0x} + k_{0y}} \sim 10^{-2}\text{–}10^{-3},$$

which, in principle, allows us to omit the term K_1 in expression (3) if we are only interested in the nature and features of the PS.

Qualitative analysis of the temperature dependence of the polarisation switching point position

Now let's get back to the basic problem of this work – polarisation switching. According to the definitions adopted in [1; 3], the PS point is determined by the value of the density of the injection current, for which the degree of polarisation of the output radiation is zero. In a steady-state approximation, this must correspond to the zero anisotropy of the gain, i. e. the condition

$$K_2 = \frac{1}{2}\left[k_{0x} - k_{0y} + j\left(k_{1x} - k_{1y}\right) + j^2\left(k_{2x} - k_{2y}\right)\right] = 0, \tag{4}$$

which is a quadratic equation, the solution of which is

$$j_{p1(2)} = \frac{-\left(k_{1x} - k_{1y}\right) \pm \sqrt{\left(k_{1x} - k_{1y}\right)^2 - 4\left(k_{2x} - k_{2y}\right)\left(k_{0x} - k_{0y}\right)}}{2\left(k_{2x} - k_{2y}\right)}. \tag{5}$$

Based on the conditions for the possibility of the existence of two points PS, we must have two real positive roots (a necessary condition), which imposes several additional conditions on the ratio of coefficients in expression (5):

1) the signs of the differences $k_{2x} - k_{2y}$ and $k_{1x} - k_{1y}$ must be opposite, otherwise at least one of the roots will be negative;

2) the positivity of both roots of the equation requires the fulfillment of the conditions $\left(k_{1x} - k_{1y}\right)^2 >$ $> 4\left(k_{2x} - k_{2y}\right)\left(k_{0x} - k_{0y}\right)$ and $4\left(k_{2x} - k_{2y}\right)\left(k_{0x} - k_{0y}\right) > 0$, which means that the signs $k_{0x} - k_{0y}$ and $k_{2x} - k_{2y}$ must be the same, with the first term always initially considered positive $k_{0x} - k_{0y} > 0$. Otherwise, you can simply override the direction of the corresponding axes (mentioned above) and the initial value for the ψ angle.

In other words, for two points of PS to exist simultaneously, the curves describing the dependencies $k_x(j)$ and $k_y(j)$ on the density of the injection current must intersect twice, as shown, for example, in fig. 1, *a*. However, this is only a necessary condition, since the obtained values of the roots (5) must still fall within the range of operating values, which is limited at the bottom by the threshold value of the current density, and at the top by the value corresponding to the thermal rollover, and this range depends significantly on the temperature [29].

There is one more important observation that needs to be made, which is important for further analysis. In the case of VCSELs, there are two types of PS, frequency reduction (type 1) and frequency upscaling (type 2) [11; 12], assuming that polarisation modes have different frequencies. In SFM models, these types of transitions are associated with different birefringence values [6; 7]. However, in the single-frequency approximation used in this paper, this classification cannot be used. At the same time, PS is a switch from one linear polarisation to an orthogonal polarisation. Within the framework of the approach under consideration, the PS points are determined by the intersection of the $k_x(j)$ and $k_y(j)$ curves, and since these are second-order curves, the type of switching point can be related to the nature of the change in the orientation of the amplitude vector of the generated radiation. In particular, if the initial orientation (after the generation threshold has been passed) is associated with the axis x, then the switch $x \to y$ can be called the first type of switch, and the reverse switch can be called the second type. Such definitions correlate well with those used in the modelling of thermal processes (see, for example, [29]), which also use a single-frequency approximation.

It's worth to introduce a few more additional restrictions on the appearance of the $k_x(j)$ and $k_y(j)$ curves, which will be convenient for subsequent analysis. Let us consider the values k_{0x} and k_{0y} to be positive, since it concerns the formal definition of the origin. Further, let's assume that k_{1m} and k_{2m} have different signs, which can be attributed to the large range of linearity of the output characteristics. Moreover, for the same reasons, k_{2m} should be considered negative, which is more consistent with the processes of thermal breakdown of generation. It is easy to see that the assumptions made are not fundamental, but they greatly facilitate the qualitative interpretation of the experimental data published in the literature.

Let's start with the results published by the authors [8] on the shift of PS points during temperature increase for laser emitters based on $Al_xGa_{1-x}As$ heterostructure emitting at a wavelength of 850 nm. The main result of this work is related to the fact that when the temperature of the emitter rises (from 273 to 323 K), the first point of the PS (according to the accepted convention, this is the transition of type 1) «slides» into the region of lower values of the injection current, and the second (transition of type 2), on the contrary, goes to the region of higher values, and the magnitude of the effect depends on the aperture of the emitter. Such an effect can be easily explained by assuming, for example, a decrease in the $k_{0x} - k_{0y}$ difference with an increase in temperature (compare curves *2* and *3* in fig. 1, *a*), which leads to some relative «vertical» shift of the k_y curve with respect to k_x, although a somewhat more complex variant is actually realised, since a change in only the value of k_{0y} will inevitably lead to a small «horizontal» shift of the k_y curve, since the maximum of the k_y curve corresponds to the value of $j_{max} = \dfrac{k_{1y}}{2k_{0y}}$. In other words, there is a rather complex variant of the k_{mn} parameter change with a change in temperature, but the presence of a dominant «vertical» shift makes it possible to qualitatively explain the observed effects [8].

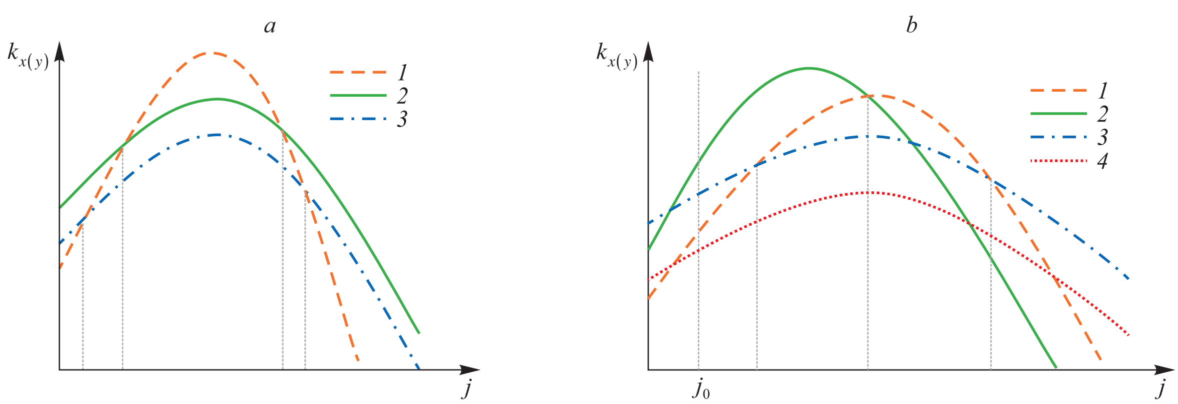

Fig. 1. The relative position of the k_x (*1*) and k_y (*2–4*) curves with increasing temperature to explain the results of [27] (*a*) and [30] (*b*). Remember, that $k_y = k_y(T)$ and $T_2 < T_3 < T_4$, where the temperature index corresponds to the number of the curve

At the same time, in order to explain the results for the same type of emitter (quantum-dimensional QW GaAs/AlGaAs VCSEL) [32], it is no longer enough to consider the dominant «vertical» shift. According to the data [32], if only the first point of PS is observed at a temperature of 10 °C (fig. 1, *b*, curve *2*), then when the temperature rises to 15 °C (fig. 1, *b*, curve *3*), the first point shifts to the area of lower current values, but a second one appears with a slightly higher value of injection current. Finally, when the temperature rises to

55 °C (fig. 1, *b*, curve *4*), the first point disappears from the operating range altogether, while the second point shifts to an even larger range. Such results can be easily explained on the basis of the phenomenological model under consideration, if we assume not only a relative «vertical» shift of the k_y curve with respect to k_x, but also a significant «horizontal» shift (see fig. 1, *b*). It should be noted that for curves *2* and *4* (see fig. 1, *b*), the second points of intersection (these are switches of the same type 1) are outside the operating range, as illustrated by the introduction of the threshold current value in fig. 1, *b*.

These examples are not proof of the adequacy of the proposed model, but rather show its potential in terms of interpreting the observed experimental results. Moreover, the data used for the analysis are in a certain sense fragmentary: the work [32] is devoted to the substantiation of the thermal mechanism of PS and the data on the temperature dependence of PS points are rather illustrative, and in the paper [8] the main emphasis is placed on the study of the multimode mode depending on the value of the transverse aperture VCSEL, in which the PS process does not have a complete form (the values of the degree of polarisation do not reach the limit values), i. e. there are both polarisation modes, but with different intensity ratios.

At the same time, the analysis shows that the availability of a sufficient amount of experimental data in principle makes it possible to calculate the approximate behaviour of k_{mn} parameters under temperature change. However, there are some doubts as to whether the behaviour of the PS points can be reduced to a relative shift of the k_x and k_y curves. To analyse this situation, let's turn to the analysis of experimental data presented in [9; 34] for long-wave VCSELs. The point is that the position of one PS point practically does not depend on the temperature of the emitter, while the second one behaves very specifically (fig. 2, *a*) – first, with an increase in temperature, the PS current value drops, then the PS point disappears altogether, and then it appears at significantly higher currents and gradually decreases to the values for the first PS. This behaviour of the second point indicates the possibility of the existence of some discontinuity region, which may arise due to the small difference between the values of k_{2x} and k_{2y} (fig. 2, *b*).

Indeed, if we assume that within a certain range of temperatures the ratio is fulfilled

$$4\left(k_{2x}-k_{2y}\right)\left(k_{0x}-k_{0y}\right) \ll \left(k_{1x}-k_{1y}\right)^2$$

for $j_{p1(2)}$ values, approximate relations can be obtained

$$j_{p1} \simeq -\frac{\left(k_{0x}-k_{0y}\right)}{\left(k_{1x}-k_{1y}\right)},\ j_{p2} \simeq \frac{\left(k_{1x}-k_{1y}\right)}{\left(k_{2x}-k_{2y}\right)} - \frac{\left(k_{0x}-k_{0y}\right)}{\left(k_{1x}-k_{1y}\right)}. \tag{6}$$

If we further assume that the parameters $k_{0x}-k_{0y}$ and $k_{1x}-k_{1y}$ change with temperature in approximately the same way, then the value of j_{p1} remains approximately constant in this interval. If we now assume that for the same temperature range the difference $k_{2x}-k_{2y}$ changes sign (i. e. passes through zero), then a range of values j_{p2} is formed that goes far beyond the range of permissible values, which is illustrated by the dependencies given in fig. 2, *b*.

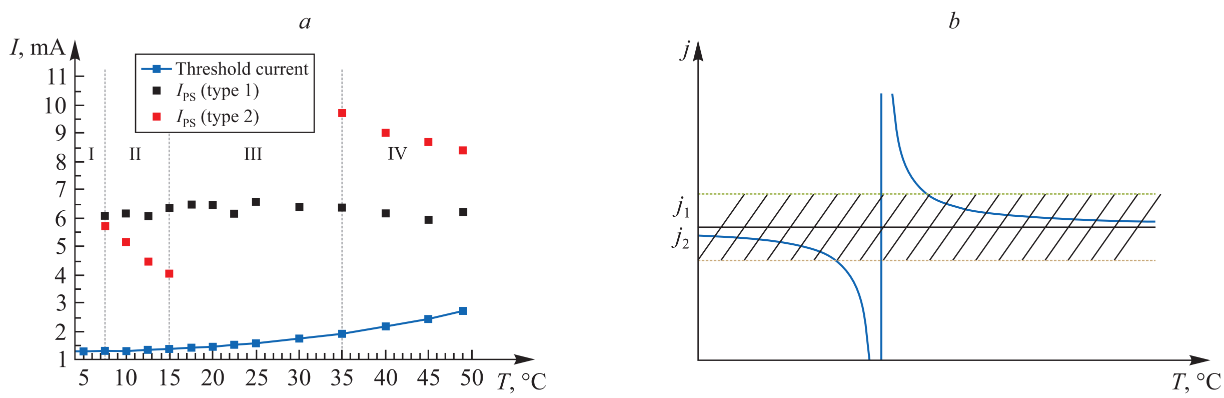

Fig. 2. Temperature dependence of PS points of different types according to the results of experimental studies by the authors [9; 34] (*a*); qualitative behaviour of the roots j_{p1} and j_{p2} in the region of the formal extinction of j_{p2} (*b*). The range of valid values for PS points is shaded

However, the situation is not as simple as it seems: according to the earlier analysis, for the simultaneous existence of two positive roots of equation (4), certain ratios of the k_{mn} coefficients must be met, i. e. a change in the sign of the difference $k_{2x}-k_{2y}$ must be accompanied by a change in the signs $k_{0x}-k_{0y}$ and $k_{1x}-k_{1y}$, although here a certain shift in the corresponding temperature values is possible (and most likely inevitable). Indeed, if we assume that all three differences turn to zero at the same temperature, then it turns out that the curves $k_x(j)$ and $k_y(j)$ for a given temperature coincide and the emitter does not exhibit anisotropy at all (a very interesting situation in itself, which may require a separate analysis). However, in this case it is impossible to preserve the position of the first point of the PS, since this is possible only when the maxima of the curves $k_x(j)$ and $k_y(j)$ are shifted, which excludes the simultaneous zeroing of $k_{2x} - k_{2y}$ and $k_{1x} - k_{1y}$. Therefore, most likely, we have a relatively narrow range of temperatures in which the signs of the differences change, which can lead to a very original picture of the distribution of the areas of existence of the solutions of equation (4), but in this situation, when it comes to the fundamental behaviour of roots, this may be the topic of a separate independent study. At the same time, the presence of discontinuities can take the PS points far beyond the range of operating values of injection currents, which is consistent with the data [9; 34] (see fig. 2, *a*) on the existence of a temperature region where only one value of PS is manifested, which corresponds to one positive root of equation (4).

The above reasoning shows that in the latter case, we can rely on the qualitative (possible) behaviour of the $k_x(j)$ and $k_y(j)$ curves to interpret the data. Indeed, let us turn to fig. 2, *a*. There are no PS points at all in area I, which means that the curves $k_x(j)$ and $k_y(j)$ do not intersect at all. Since this analysis is qualitative, for the sake of simplicity, we will assume that the position of the $k_x(j)$ curve does not change with changes in temperature. In this case, on the border of areas I and II in fig. 2, *a*, the position of the two points of the PS must coincide, which means that the curves $k_x(j)$ and $k_y(j)$ touch each other at the same point (fig. 3, *a*, curves *1* and *2*), and on the basis of the above considerations, it should be assumed that the curve $k_y(j)$ has a greater curvature. As the temperature increases, the maximum of the $k_y(j)$ curve shifts to the region of lower currents, and the curve itself decreases the curvature due to the increase in the parameter k_{2y} (fig. 3, *a*, curve *3*). In this case, the initial touch point is retained as the position of the PS point of the type 2, but the PS point of the type 1 appears when the injection current value is lower. As the $k_y(j)$ curve is sequentially deformed, the PS point of the type 1 rapidly shifts to the region below the lasing threshold. In the case where k_{2y} is close to the value of k_{2x}, the position of this point tends to infinity, the sign of which is determined by the difference sign. Therefore, in the region $k_{2x} - k_{2y} \sim 0$ we get practically «parallel» parabolas, the shift of which is determined by the difference $k_{0x} - k_{0y} < 0$, and the position of the maximum of the curve $k_y(j)$ is shifted to the region of smaller currents, provided that in the same region $k_{1x} - k_{1y} \sim 0$ (fig. 3, *a*, curve *4*).

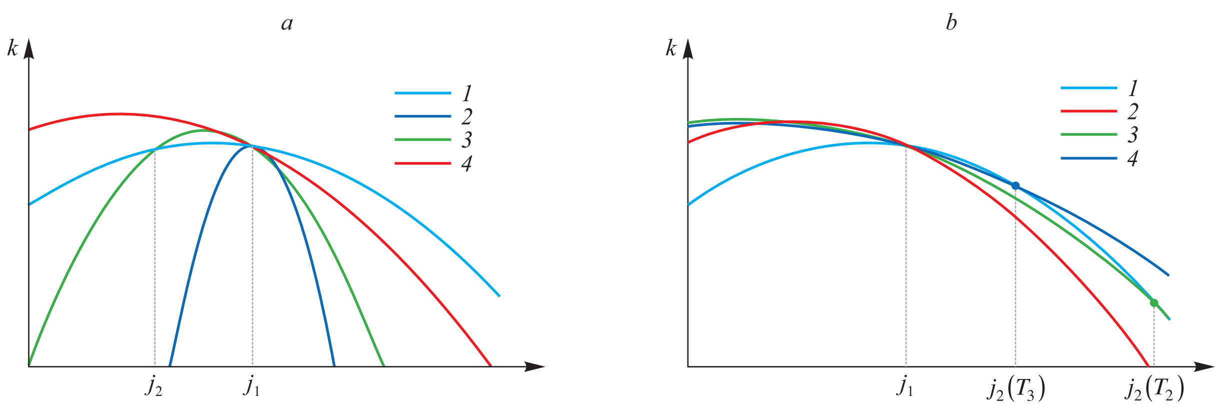

Fig. 3. Relative position of the k_x (*1*) and k_y (*2–4*) curves with increasing temperature for the region $k_{2x} - k_{2y} < 0$ (*a*) and $k_{2x} - k_{2y} > 0$ (*b*). Curve *4* corresponds to the condition $k_{2x} - k_{2y} \approx 0$, and $k_y = k_y(T)$ and $T_2 < T_3$, with the temperature index corresponding to the curve number

Now let's look at what happens after the change of sign from $k_{2x} - k_{2y}$ (fig. 3, *b*). Now the $k_y(j)$ curve has a lower curvature, and the PS point of the type 1 is shifted to the area of higher pump current values than for the PS point of the type 2 (see fig. 3, *b*, curve *4*). Characteristically, a decrease in the value of k_{2y} leads to a shift of this point to the region of larger values of currents.

Thus, the behaviour of the PS point of the type 1 finds a perfectly acceptable explanation through the behaviour of the curves $k_x(j)$ and $k_y(j)$, and the whole question is whether the stability of the PS point of the type 2 can be explained from the same point of view, since if we analyse solution (6), then the change of signs of the differences $k_{0x} - k_{0y}$ and $k_{1x} - k_{1y}$ must occur, if not at the same temperature, at least within a narrow temperature range. However, experimental studies [9; 34] have not revealed any peculiarities in the behaviour of the stable point of PS, which may be the result of two main reasons. First of all, the temperature range is large enough for two consecutive measurements, which makes it possible to «skip» special points, since the authors [9; 34] were not looking for anything like this. And secondly, and this can be considered a more significant reason, PSs occur in areas of significant polarisation instability, and statistical phenomena can play an important role, which can simply level out the features of the polarisation rearrangement process. This explanation is also supported by the statistical spread of PS currents within about 10 % (such data are not given in [9; 34], so the assessment was carried out visually based on experimental data).

These examples show that the proposed model is quite workable, although it is quite clear that the procedure for determining k_{mn} coefficients as a function of temperature may be quite complex and require a large amount of experimental data. However, the relatively small value of k_{mn} allows us to hope that an approximate qualitative description can also be quite successful.

Another interesting fact to note about the case presented in fig. 3, *b*, is the convergence of the PS points with the increase in temperature. This can be interpreted as an increase in the influence of the quadratic term in relation (2), which can lead to a corresponding change in the sign of the rate of change of the relative anisotropy with an increase in the injection current. This, in turn, can lead to what the authors [35] call «negative polarisation hysteresis», although in this case this is nothing more than an assumption that requires further verification.

Conclusions

Our investigations show that the phenomenological dependence of the induced anisotropy of the gain and (or) losses in the form of a second-order polynomial on the degrees of density of the injection current, where the polynomial coefficients are functions of temperature, proposed on the basis of the analysis of the available theoretical and experimental data, turned out to be quite successful not only from the point of view of the physical nature of the process of polarisation switching in VCSELs, but also from the point of view of the qualitative explanation of the experimental data available in the literature on the temperature dependencies of the position of PS points for various types of emitters. Practically the entire model is based on the sequential change of the $k_y(j)$ curve relative to the $k_x(j)$ curve with increasing temperature. It is in this sense that it is clear that the assumptions made about the behaviour of polynomial coefficients with temperature change are aimed only at describing the entire series of experimental results. At the same time, it should be noted that the assumptions made do not have internal contradictions.

Of course, it would be important to conduct a direct simulation of the influence of thermal processes on the polarisation dependencies of output characteristics based on our model of polarisation components, but such modelling lacks one essential component – understanding the complex of physical processes that determine the mechanisms of anisotropic response formation of a semiconductor structure. The reverse procedure seems to be more realistic, i. e. an approximate calculation of polynomial expansion coefficients based on the analysis of a set of experimental data. However, it should be borne in mind that a single-mode approximation is used in this work, and if there are several modes, which is the case, for example, for wide-aperture VCSELs [36], then the question immediately arises regarding their independence and spectral separation due to the complex spectral-polarisation composition of the generated radiation [37]. Such a formulation of the question is quite legitimate, as evidenced, for example, by the results of work [36], where at least six PS points are observed for the multimode mode, but all these PS have the character of the concept of an incomplete cycle introduced above, despite the stationary nature of the emitter excitation.

However, the main conclusion of the work is not even related to the proposed qualitative model, but to the clarity of the physical interpretation of the observed effects based on the approach developed by the authors based on the method of polarisation components. It should be noted that the SFM model, which is very popular for VCSEL, was developed for quantum-dimensional semiconductor structures [6] in the steady-state excitation approximation, while for the method used in the work, the main limitation is the presence of rapid phase relaxation. Therefore, it is quite reasonable to expect that this technique will be suitable for studying polarisation effects in emitters based on short-period superlattices [38].

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