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ОБРАЩЕНИЕ НАПРАВЛЕНИЯ ДВИЖЕНИЯ БРОУНОВСКОГО РЭТЧЕТА, НАВЕДЕННОЕ ОБРАЩЕНИЕМ КРИВИЗНЫ РЭТЧЕТ-ПОТЕНЦИАЛА

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Аннотация. Исследуется броуновское движение частицы в стационарном потенциальном профиле универсальной симметрии, дрейф которой происходит вследствие дихотомных флуктуаций этого профиля. Координатная зависимость дихотомных флуктуаций описывается симметричной или антисимметричной периодическими функциями. В отличие от существующих моделей рэтчет-систем асимметрия рассматриваемой системы вносится флуктуациями за счет сдвига положений осей или центров их симметрии относительно стационарного профиля. В высокотемпературном приближении получены аналитические соотношения для частотной зависимости средней скорости рэтчета, из которых следует наличие рэтчет-эффекта при учете двух пространственных гармоник флуктуационного потенциала. Температурные зависимости средней скорости рассчитаны в адиабатическом режиме движения. Показано, что обращение знака кривизны стационарной составляющей потенциального профиля

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при пространственно симметричных флуктуациях может приводить к обращению направления движения. Проанализированы зависимости средней скорости от частоты флуктуаций, сдвига осей симметрии стационарного и флуктуирующего вкладов в потенциальную энергию, а также от отношений амплитуд этих вкладов к тепловой энергии. Выявлены оптимальные режимы движения рассматриваемых рэтчет-систем.

Ключевые слова: броуновский рэтчет; диффузионный транспорт; неравновесные флуктуации; дихотомный процесс; периодический потенциал; универсальная симметрия.

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REVERSAL OF THE MOTION DIRECTION OF A BROWNIAN RATCHET INDUCED BY THE CURVATURE REVERSAL OF THE RATCHET POTENTIAL

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Abstract. Brownian motion of a particle in a stationary potential profile of universal symmetry is considered, the drift of which occurs due to dichotomous fluctuations of that profile with the coordinate dependence of the fluctuations described by either symmetric or antisymmetric periodic functions. Unlike existing models of ratchet systems, the asymmetry of the system under consideration is introduced by means of fluctuations due to a shift in the positions of their symmetry axes or symmetry centers relative to the stationary profile. In the high-temperature approximation, analytical relations are obtained for the frequency dependence of the ratchet average velocity, which implies the existence of the ratchet effect when two spatial harmonics of the fluctuating potential are taken into account. The temperature dependences of the average velocity have been calculated in the adiabatic mode of the motion. It is shown that the reversal of the curvature sign of the stationary component of the potential profile under spatially symmetric fluctuations can lead to a reversal of the motion direction. The dependences of the average velocity on the fluctuation frequency, the shift of the symmetry axes of the stationary and fluctuating components of the potential energy, as well as on the ratio of the amplitudes of these components to the thermal energy have been analysed. Optimal motion modes of the ratchet systems under consideration have been revealed.

Keywords: Brownian ratchet; diffusion transport; nonequilibrium fluctuations; dichotomous process; periodic potential; universal symmetry.

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Introduction

Ratchet systems are asymmetric systems that, being in contact with a thermostat, are capable of generating a directed motion in the absence of stationary external forces, when consume energy from nonequilibrium fluctuations of various natures [1–9]. Among the variety of approaches to the description of such systems, a special place is occupied by theoretical researches based on the consideration of the diffusion dynamics of a single particle in a time-dependent periodic potential field $U(x, t)$ (x is the particle coordinate and t is time). Such approaches are the simplest, since, in them, a single Brownian particle or an ensemble of non-interacting particles acts as the main object of consideration, but, at the same time, they are the most instructive, since they allow one to clarify the basic conditions for the emergence of the directed motion [1; 3; 4; 7–9]. The ratchet effect, i. e. the phenomenon of the emergence of the directed motion, is possible for functions $U(x, t)$ that are free from a number of symmetry prohibitions, which are governed by the symmetry theory of such systems [10; 11]. In addition to the obvious prohibition on the mirror symmetry, there are also hidden symmetries that are «hidden» in the general properties of the solutions to the Smoluchowski equation [12–14].

To classify ratchet systems, the function $U(x,\ t)$ is frequently considered as having an additive-multiplicative form $U(x,\ t) = u(x) + \sigma(t)w(x)$, in which the functions of the coordinate $u(x)$ and $w(x)$ describe the stationary and fluctuating contributions to the potential energy respectively while the function $\sigma(t)$ describes the time dependence of the fluctuations. This functional form of the particle potential energy allows for distinguishing two main classes of ratchet systems: ratchets with a fluctuating tilting homogeneous force F (rocking ratchets), for which $u(x)$ is a spatially periodic function and $w(x) = -Fx$, and ratchets with the fluctuating periodic potential profile (pulsating or flashing ratchets), for which $u(x)$ and $w(x)$ are arbitrary spatially periodic functions [3; 8; 9]. Symmetric dichotomous fluctuations are frequently considered, in which the function $\sigma(t)$ takes two values, i. e. +1 and −1. Dichotomous processes differ in the nature of the alternating the states. If the potential profiles are switched with time intervals $\frac{\tau}{2}$, then we speak of a symmetric deterministic periodic dichotomous process with the period τ. If the transitions between values ± 1 occur at random moments in time, then we speak of a stochastic dichotomous process. The deterministic nature of processes is typical for artificially created ratchets [4; 5; 7], the stochastic nature is mostly associated with ratchets of natural origin (acting in living and nonliving objects) [1; 3; 6; 7]. The common property of these two symmetric processes is the zero mean value $\langle \sigma(t) \rangle = 0$. The deterministic process is characterised by the period τ, while the stochastic process by the inverse correlation time Γ, which is determined by the correlation function of the dichotomous process: $\langle \sigma(t)\sigma(t') \rangle = \exp(-\Gamma|t - t'|)$ [15; 16].

The properties of ratchet systems depend significantly on the values of two dimensionless parameters: the ratio of the potential barrier ΔU to the thermal energy $k_{\mathrm{B}}T$ (k_{B} is the Boltzmann constant, T is absolute temperature) and the ratio of the period τ or the correlation time Γ^{-1} to the characteristic relaxation time τ_{rel}, which, at $\frac{\Delta U}{k_{\mathrm{B}}T} \ll 1$, can be estimated by the characteristic diffusion time $\tau_D = \frac{L^2}{D}$ ($D = \frac{k_B T}{\zeta}$ is the diffusion coefficient, ζ is the friction coefficient) on the spatial period L of the potential $U(x,\ t)$, and, at $\frac{\Delta U}{k_{\mathrm{B}}T} \gg 1$, by the time $\tau_{\mathrm{sl}} = \frac{\zeta L^2}{\Delta U}$ of particle sliding down the potential barrier ΔU. At $\tau \gg \tau_{\mathrm{rel}}$ (or equivalently $\Gamma\tau_{\mathrm{rel}} \ll 1$), in each of the states of the symmetric dichotomous process, thermodynamic equilibrium has time to be established to the moment of switching the states (adiabatic ratchet). In this case, the average velocity of a rocking ratchet does not depend on the fluctuation frequency (τ^{-1} or Γ), while for a pulsating ratchet, it is proportional to the fluctuation frequency. As the fluctuation frequency increases, the average velocity of the rocking ratchet decreases, and the pulsating ratchet velocity first increases to its maximum value at $\tau \sim \tau_{\mathrm{rel}}$ (or $\Gamma\tau_{\mathrm{rel}} \sim 1$), and then also decreases [8; 17].

At $\frac{\Delta U}{k_{\mathrm{B}}T} \ll 1$, the motion of a Brownian particle is of a diffusion nature, for which the behaviour of the particle in the wells of the potential relief is important, and the high-temperature approximation is applicable to describe the motion [15; 16]. Ratchets operate most effectively when the potential profile with $\frac{\Delta U}{k_{\mathrm{B}}T} \gg 1$ fluctuates by half a period [18]. At $\frac{\Delta U}{k_{\mathrm{B}}T} \gg 1$, the main contribution to the average ratchet velocity is made by the hopping motion of the particle associated with overcoming the barriers that separate the potential wells. The description becomes discrete and can be carried out in terms of the velocities of overcoming the potential barriers or the probabilities of transitions between the discrete positions of the particle in space [19]. The advantage of the discrete description over the continuous one is that it can be applied beyond the diffusion dynamics, since the transition probabilities included in the expressions for the average ratchet velocity can also be calculated for transitions between the states of quantum particles. This makes it possible to take into account the contribution of tunneling processes to the average ratchet velocity, which can even reverse the motion relative to the motion that occurs solely due to the thermally activated processes [20–23].

At the beginning of the development of ideas about the mechanisms of functioning of Brownian motors (ratchets), the simplest models were considered in which an asymmetric sawtooth potential was disturbed by either a dichotomously fluctuating external force or the fluctuating potential barrier, which led to the motions in opposite directions [17]. If in one of the states of the dichotomous process, the barrier is absent [24; 25], then the average velocities, which are opposite in direction, of such a pulsating ratchet (called in this case «an on-off ratchet») and a ratchet with the small fluctuating force, even when we include small inertial effects, are determined by the same factor (which is the same for the two ratchets), that depends on the parameters of the stationary potential [26; 27].

Further studies of ratchet systems revealed a number of new their properties caused by the contribution of the two different functions $u(x)$ and $w(x)$ to the total potential energy $U(x,\ t)$. Each of these functions may have its own symmetry elements, the set of which determines the unique symmetry properties of the ratchet under consideration. Periodic functions can be symmetric, antisymmetric, or shift-symmetric. The first two symmetry properties are given by the equalities $u(x) = \pm\, u(-x + 2x_{\pm})$, where the upper and lower signs correspond to the symmetry with the axis x_+ or antisymmetry with the center x_-. Shift symmetry implies the property $u(x) = -u\left(x + \frac{L}{2}\right)$, where L is the period of the function $u(x)$. For example, if we describe ratchet systems within the framework of the Smoluchowski equation and do not include inertial effects, then the ratchet effect is absent for supersymmetric potential energies with the following property: there exists time t_s such that $U(x,\ t) = -U\left(x + \frac{L}{2},\ -t + 2t_s\right)$ [10]. For functions that simultaneously satisfy the above two symmetry properties, the third one is always satisfied, and the symmetry axis and symmetry center closest to each other are separated by the distance of $\frac{L}{4}$, $|x_+ - x_-| = \frac{L}{4}$. Such functions are classified as functions of universal symmetry [13; 14].

The simplest function of universal symmetry is a sinusoid. If the coordinate dependence of the fluctuating part of the potential energy has such a shape, i. e. $w(x) = w\cos 2\pi\left(\frac{x}{L} - \lambda_0\right)$, then we speak of a spatially harmonic governing signal with the amplitude w and phase shift λ_0 [16]. Such a signal is most easily realised experimentally and makes it possible to control the magnitude and direction of the average ratchet velocity by changing the parameter λ_0. At the faculty of physics of Belarusian State University, a number of theoretical results have been obtained regarding the properties of ratchet systems with an asymmetric sawtooth and stepwise stationary potential $u(x)$, that operate being dichotomously perturbed by a spatially harmonic signal [28; 29]. In this article, we consider the opposite case: the stationary contribution $u(x)$ to the potential energy is characterised by the universal type of symmetry, while asymmetry is introduced into the system by the spatial dependence of the fluctuating contribution $w(x)$, that can be realised by the sum of two spatial harmonics.

The possibility of such control of the directed motion arises in ratchets on optical lattices, in which both the periodic stationary contribution $u(x)$ and the fluctuation contribution $w(x)$ are determined by laser fields, which allow creating arbitrary spatial asymmetry [30–32]. The high-temperature approximation used in the next section of this paper allows the ratchet average velocity to be written as a series in spatial harmonics of the functions $u(x)$ and $w(x)$. From such a representation, it immediately follows that, for the function $u(x)$ of the universal symmetry type, the possibility of the ratchet effect is realised already by the first two harmonics of the function $w(x)$. In addition, an analysis of the obtained representation shows that a step-shaped function $u(x)$ forbids the ratchet effect only if the symmetry axis of the function $w(x)$ coincides with the symmetry center of the function $u(x)$. The deformation of the step allows for the ratchet effect, and the motion can be reversed by the change in the sign of the curvature of the function $u(x)$.

While the high-temperature approximation allows for the analysis of the frequency dependence of the average velocity, the adiabatic approximation allows for the low-frequency picture of the temperature dependence.

High-temperature approximation and frequency dependences

The high-temperature approximation is understood as a simplification of equations and their solutions under the assumption that the thermal energy significantly exceeds the barriers of the periodic potential profiles. The first use of such an approximation in the theory of ratchets was carried out in [15]. In the most general form, the average velocity of a high temperature ratchet with an arbitrary time dependence of its potential energy of the additive-multiplicative form $U(x,\ t) = u(x) + \sigma(t)w(x)$ can be represented as

$$\begin{gathered} v = iD^2\beta^3 \sum_{\substack{q,\, q'(\neq 0) \\ (q + q' \neq 0)}} k_q k_{q'} k_{q+q'} w_q w_{q'} u_{-q-q'} \Psi\left(Dk_q^2,\, Dk_{q'}^2\right), \\ \Psi(a,\ b) = \frac{1}{a-b}\int_0^\infty dt\, K_2(t)\left(ae^{-at} - be^{-bt}\right),\ \ K_2(t) = \left\langle \sigma(t')\sigma(t'-t)\right\rangle, \end{gathered} \tag{1}$$

where $k_q = \dfrac{2\pi q}{L}$ (q is an integer), and the time dependence of the fluctuations forms the frequency dependence of the average velocity by means of the pair correlation function $K_2(t)$.

While the symmetric stochastic dichotomous process is characterised by an exponential decay of the correlation function with the inverse correlation time Γ, the correlation function of the deterministic dichotomous process is a symmetric sawtooth function of time, described on the period of its change by the equation $K_2(t) = 1 - \dfrac{4|t|}{\tau}$, $|t| \le \dfrac{\tau}{2}$. Note that for large periods τ, both correlation functions have linear asymptotics, which become identical at $\Gamma = \dfrac{4}{\tau}$. This means that the stochastic and deterministic processes become indistinguishable when the period (average period) of the process is much longer than the relaxation time. It is the function $\Psi\left(Dk_q^2, Dk_{q'}^2\right)$ that determines the competition of the process period with the inverse relaxation time $Dk_1^2 = \dfrac{4\pi^2}{\tau_D}$. For the stochastic and deterministic dichotomous processes, this function is given by the following expressions:

$$\Psi(a,\ b) = \frac{\Gamma}{(\Gamma + a)(\Gamma + b)},\ \ \Psi(a,\ b) = \frac{4}{(a-b)\tau}\left(\frac{1}{b}\tanh\frac{b\tau}{4} - \frac{1}{a}\tanh\frac{a\tau}{4}\right).$$

Both expressions vanish, i. e. the ratchet effect is absent, at both zero and infinite fluctuation frequencies. While the low-frequency asymptotics of these expressions are linear and identical at $\Gamma = \dfrac{4}{\tau}$, $\Psi(a,\ b) \approx \dfrac{\Gamma}{ab}$, their high-frequency asymptotics are different, namely, proportional to Γ^{-1} and τ^2 for the stochastic and deterministic processes respectively. Therefore, the frequency dependences of the ratchet average velocity are described by bell-shaped functions, the width of which is noticeably larger for stochastic processes than for deterministic ones.

Let us now consider the spatial dependences of fluctuations that are described by periodic functions belonging to the class of either symmetric function $w_+(x)$ or antisymmetric function $w_-(x)$, whose Fourier components satisfy the relations $w_{\pm,\,q} = \tilde{w}_{\pm,\,q}\exp\left(-ik_q x_\pm\right)$, where $\tilde{w}_{\pm,\,q} = \pm\tilde{w}_{\pm,\,-q}$ are the Fourier components of the even or odd function $\tilde{w}_\pm(x) = w_\pm(x + x_\pm)$. For the function $u(x)$ of universal symmetry with the symmetry center at a point x_a, $u(x) = -u(-x + 2x_a)$, its shift symmetry leads to the vanishing of all its Fourier components u_q with even q, and the expression (1) can be represented as

$$v = -4iD^2\beta^3 \sum_{l,\,l'=1}^{\infty} k_{2l}k_{2l'-1}\Psi\left(Dk_{2l}^2,\ Dk_{2l'-1}^2\right)\tilde{w}_{\pm,\,2l}\tilde{w}_{\pm,\,2l'-1} \times$$
$$\times\left[k_{2l+2l'-1}u_{2l+2l'-1}\cos\left(k_{2l+2l'-1}x_0\right) \mp k_{2l-2l'+1}u_{2l-2l'+1}\cos\left(k_{2l-2l'+1}x_0\right)\right], \tag{2}$$

where $x_0 = x_\pm - x_a$.

Several important conclusions follow from this result. Since $\cos\left(k_p x_0\right)$ is an even function of x_0 and $\cos\left(k_p\left(x_0 + \dfrac{L}{2}\right)\right) = -\cos\left(k_p x_0\right)$ at odd p, the ratchet average velocity as a function of x_0 satisfies the relations

$$v(x_0) = -v\left(x_0 + \frac{L}{2}\right) = v(-x_0),$$

and, therefore, is a function of universal symmetry with respect to the difference in the coordinates of the locations of the symmetry axis or symmetry center of the function $w_\pm(x)$ and the symmetry center of the function $u(x)$. Thus, for fluctuations whose spatial dependence is described by either symmetric or antisymmetric function, the universal symmetry of the function $u(x)$ has led to the universal symmetry of the function $v(x_0)$ itself. From this, in particular, follows the absence of the ratchet effect when the distance between the symmetry axis or symmetry center of the function $w_\pm(x)$ and the symmetry center of the function $u(x)$ is a quarter of the period ($|x_0| = \dfrac{L}{4}$). For a symmetric function $w_+(x)$, this result is obvious, since under this condition, the potential energy $U(x,\ t)$ becomes a symmetric function of x. For an antisymmetric function $w_-(x)$, the absence of the ratchet effect is a consequence of the universal symmetry of the function $\sigma(t)$.

Let us consider next a deformed symmetric stepwise potential of the universal symmetry type, characterised by the step height $2u$ and a curvature α (fig. 1). Its shape on the period L and nonzero Fourier components are given by the following formulas:

$$u(x)=\begin{cases} u+\dfrac{1}{2}\alpha x\left(x-\dfrac{L}{2}\right), \ 0\le x\le\dfrac{L}{2}, \\ -u+\dfrac{1}{2}\alpha(L-x)\left(x-\dfrac{L}{2}\right), \ \dfrac{L}{2}\le x\le L, \end{cases} \tag{3}$$

$$u_{2l-1}=-\frac{4i}{k_{2l-1}L}\left[u-\frac{\alpha}{k_{2l-1}^2}\right].$$

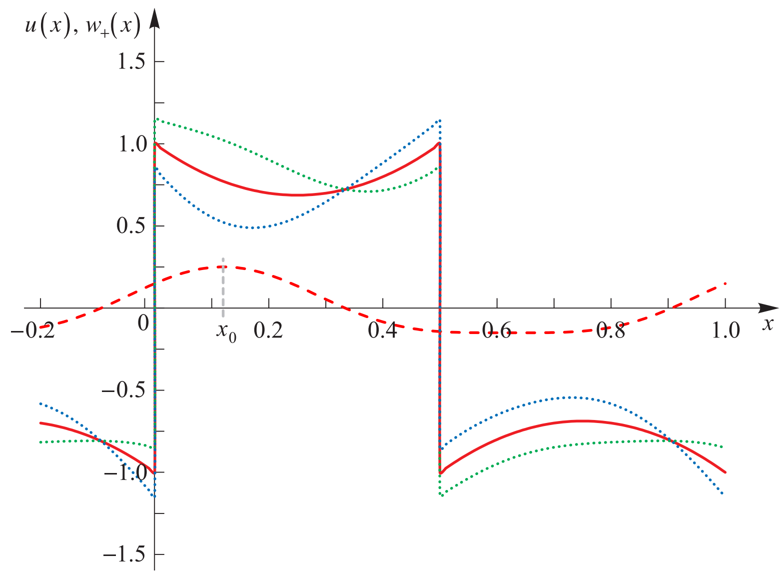

Fig. 1. Stationary contribution $u(x)$ of the universal symmetry type, represented by the deformed stepwise potential (3) (the solid line), fluctuating contribution described by the symmetric function $w_+(x)$ (the dashed line), and the total potential energy in two states of the dichotomous process (the dotted lines). Energy quantities are measured in units of u, lengths are measured in units of L

Substituting the Fourier components from (3) into the expression (2) leads to the result

$$v(x_0)=\frac{16}{L}D^2\beta^3u\sum_{l,\,l'=1}^{\infty}k_{2l}k_{2l'-1}\Psi\left(Dk_{2l}^2, Dk_{2l'-1}^2\right)\tilde{w}_{\pm,\,2l}\tilde{w}_{\pm,\,2l'-1}f_{\pm,\,2l,\,2l'-1}(x_0),$$

$$f_{\pm,\,q,\,q'}(x_0)=\frac{\alpha}{u}\left(\frac{\cos k_{q+q'}x_0}{k_{q+q'}^2}\mp\frac{\cos k_{q-q'}x_0}{k_{q-q'}^2}\right)-\left(\cos k_{q+q'}x_0\mp\cos k_{q-q'}x_0\right), \tag{4}$$

from which a number of important and interesting conclusions follow. For symmetric function $w_+(x)$, at $x_0=0$, the ratchet effect is absent for a symmetric stepwise potential $u(x)$, but appears under the deformation of $u(x)$ that makes its curvature α nonzero. Unlike ordinary stopping points, passing through which is accompanied by the motion reversal, this stopping point $x_0=0$ corresponding to $\alpha=0$ is characterised by the same sign of the velocity on both sides of it. For small nonzero x_0 and $\alpha\ne0$,

$$f_{+,\,q,\,q'}(x_0)\approx 2k_qk_{q'}\left(x_0^2-\frac{2\alpha}{uk_{q+q'}^2k_{q-q'}^2}\right). \tag{5}$$

Therefore, when $\alpha<0$, there are no stopping points near $x_0=0$, while for $\alpha>0$, two closely located ordinary stopping points appear, the locations of which are determined by the zeros of the functions $f_{+,\,q,\,q'}(x_0)$.

Since the locations of these zeros depend on the summation indices, and the function (5) with particular q and q' is not the general multiplier for all terms in the sum, the zeros of the average velocity can become frequency dependent due to the presence of functions $\Psi\left(Dk_q^2, Dk_{q'}^2\right)$ in the expression (4).

For symmetric fluctuations described by the sum of two cosine waves

$$w(x) = w\left[\cos\left(2\pi\left(x - x_0\right)\right) + \frac{1}{4}\cos\left(4\pi\left(x - x_0\right)\right)\right], \tag{6}$$

the summation in (4) is limited to one term with $l = l' = 1$, so that the dependence $v(x_0)$ is determined by the function $f_{+,2,1}(x_0)$, which is depicted in fig. 2.

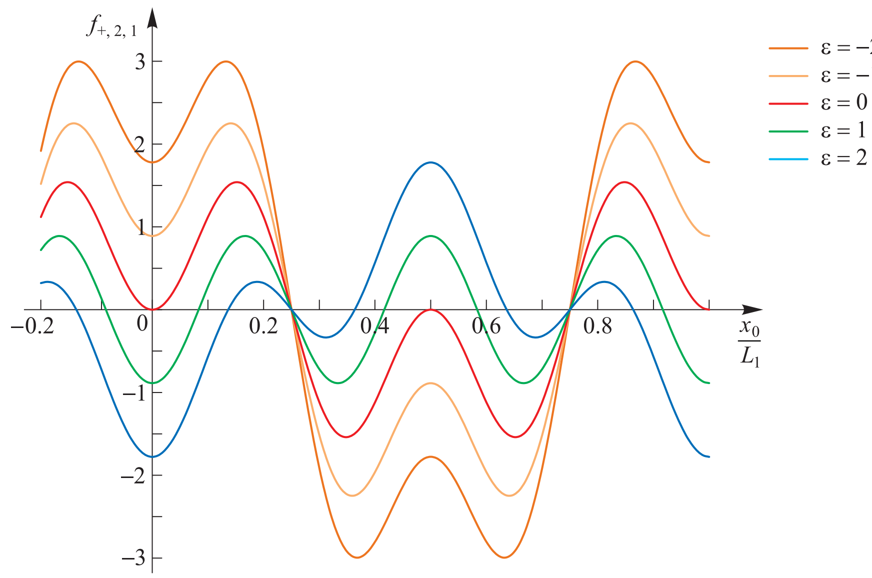

Fig. 2. Dependences of the average velocity of the high-temperature ratchet, in dimensionless units (factor $f_{+,2,1}(x_0)$ in equation (4)) on the shift of the symmetry axes of $w_+(x)$ relative to the symmetry center of the deformed stepwise function $u(x)$ at different values of the deformation parameter $\varepsilon = \frac{\alpha L^2}{4\pi^2 u}$ (the values near the curves)

The symmetry axis and symmetry center of the function $f_{+,2,1}(x_0)$ pass through the points $x_0 = 0$ and $\frac{L}{4}$ respectively, so that this function indeed belongs to the universal symmetry type. For the negative curvature of the deformation of the stationary stepwise profile, the zeros of the function $f_{+,2,1}(x_0)$ coincide only with the symmetry axes of the function $u(x)$, whereas for the positive curvature, additional paired stopping points arise. In the next section, the effect of the reversal of the ratchet motion when the curvature of the stationary part $u(x)$ of the potential relief, undergoing symmetric dichotomous fluctuations in time, is reversed will be described in detail applying the adiabatic approximation.

Low-temperature approximation and temperature dependences

The adiabatic mode of motion implies the absence of heat exchange between the system and the environment[1]. It can be implemented in two ways: by either slow or instantaneous change in the set of parameters that determine the shape of the spatially periodic potential. A ratchet with dichotomous switching the states with different potential profiles (pulsating ratchet) is called an adiabatically fast driven ratchet if the lifetime of each state is so long that the thermodynamic equilibrium has time to be established in them [33]. In terms of the fluctuation frequency, adiabatically fast ratchets are described by the low-frequency approximation.

According to famous Parrondo's lemma [7], the average velocity v of a Brownian particle, that resides in two cyclically switching long-lived states characterised by periodic potential reliefs $U(x, t) = u(x) + \sigma(t)w(x)$, $\sigma(t) = \pm 1$,

[1]*Landau L. D., Lifshitz E. M.* Statistical physics. Part 1 / transl. from the Russ. by J. B. Sykes, M. J. Kearsley. 3rd ed., rev. and enl. Oxford : Pergamon Press, 1980. XVIII, 544 p. (Course of theoretical physics ; vol. 5).

is determined by the flux Φ through an arbitrary cross-section over a time interval equal to the period of the dichotomous process [33]:

$$v = \frac{L}{\tau}\Phi,\ \Phi = \int_0^L dx\left[q_+(x) - q_-(x)\right]\int_0^x dy\left[\rho_+(y) - \rho_-(y)\right],$$
$$q_\pm(x) = \frac{\exp\left[u(x) \pm w(x)\right]}{\int_0^L dz \exp\left[u(z) \pm w(z)\right]},\ \rho_\pm(x) = \frac{\exp\left[-u(x) \mp w(x)\right]}{\int_0^L dz \exp\left[-u(z) \mp w(z)\right]} \tag{7}$$

(here and below, the energy characteristics are measured in units of thermal energy $k_B T$). While the high-temperature approximation allows one to obtain frequency dependences of the average velocity, the low-frequency approximation yields temperature dependences. The effect of motion reversal with the reversal of the curvature of the stationary contribution to the potential energy will be analysed for the function $u(x)$ of the form (3) and for symmetric fluctuations described by the sum of two cosine curves (6). Numerical integration of expressions (7) with the potential energy components of the form (3) and (6) leads to the dependences presented in fig. 3 and 4.

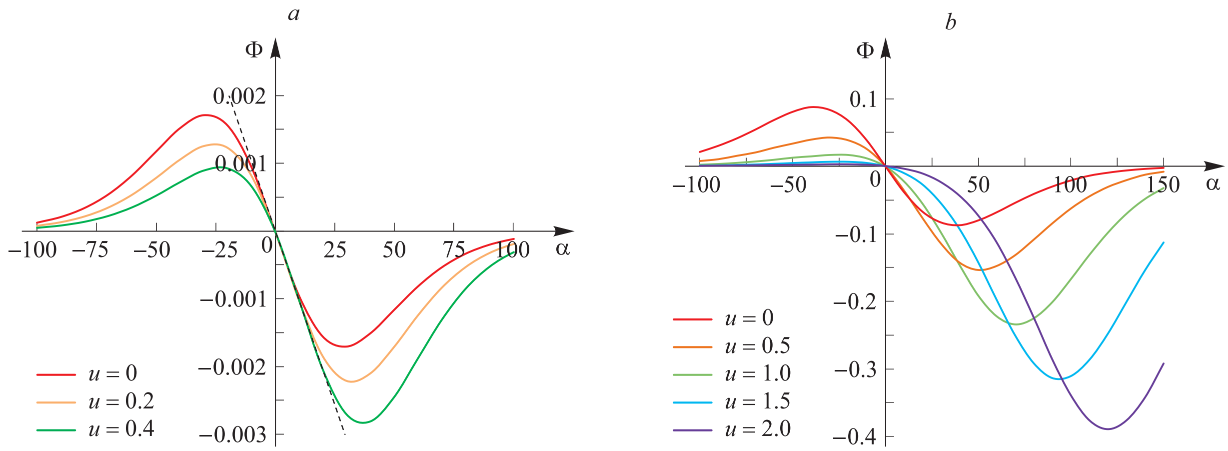

Fig. 3. Particle flux Φ through an arbitrary cross section per time equal to the period of the dichotomous process as a function of the curvature α of the deformed stepwise potential given by the formula (3) without a shift of the symmetry axis of $w_+(x)$ relative to the symmetry center of $u(x)$, $x_0 = 0$ (the energy characteristics are measured in units of thermal energy $k_B T$ and the lengths are measured in units of L): high-temperature motion mode, small fluctuations with $w = 0.3$ (the dashed line corresponds to the asymptotic behaviour given by equation (8)) (*a*); large-fluctuation motion mode with $w = 3$ (*b*)

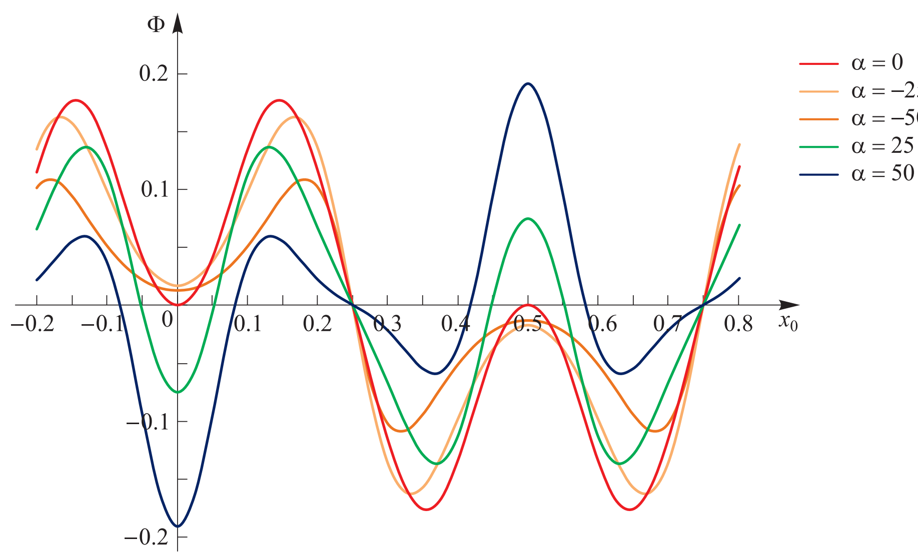

Fig. 4. Particle flux Φ through an arbitrary cross section per time equal to the period of the dichotomous process as a function of the shift x_0 of the symmetry axis of $w_+(x)$ relative to the symmetry center of $u(x)$ for $u = 1$, $w = 3$, and different values of the curvature α (the units of measurement are the same as in fig. 3)

For small values of the amplitudes of the functions $u(x)$ and $w(x)$ relative to the thermal energy $k_B T$ (high-temperature approximation), the flux (7) in the adiabatic motion mode becomes independent of the step height $2u$ at $x_0 = 0$:

$$\Phi \approx \frac{-1}{\left(4\pi^2\right)\beta^3 w^2 \alpha L^2}. \tag{8}$$

Due to this, for small α, the family of functions $\Phi(\alpha)$ (7) with different u coincides with the linear dependence (8) (see dashed line in fig. 3, *a*). If the amplitudes of function $w(x)$ are not small, the values of the flux become different with a change in u (see fig. 3, *b*). The sign of the function $\Phi(\alpha)$ reverses when the curvature α passes through zero, and the dependence $\Phi(\alpha)$ has a non-monotonic character and tends to zero as $\alpha \to \pm\infty$, since infinite potential barriers arise for these limits, which are insurmountable by the particle. With an increase in the step height or a decrease in temperature (parameter u), the positive values of the flux decrease, while the negative values increase in absolute value.

Figure 4 depicts a family of functions $\Phi(x_0)$ for different α. All curves pass through zero values at $x_0 = \frac{1}{4}$ and $x_0 = \frac{3}{4}$, that correspond to the symmetry prohibition of the ratchet effect at the coincidence of the symmetry axes of the stationary and fluctuating parts of the potential energy. For a positive curvature ($\alpha > 0$) of the deformed stepwise function $u(x)$, four additional stopping points of the ratchet arise on the period of change of $\Phi(x_0)$, which degenerate into two points ($x_0 = 0$ and $x_0 = \frac{1}{2}$), when $\alpha = 0$, and which disappear for negative curvature ($\alpha < 0$). Thus, by means of reversing the curvature of the potential profile, one can control the appearance of additional stopping points. Comparison of the curves in fig. 4 shows that, as the temperature decreases (the curvature values increase in absolute value), the magnitude of the flux decreases for negative curvature and increases for positive curvature. Such an effect was absent for the high-temperature mode in fig. 2.

Discussion and conclusions

The ratchet effect, as the phenomenon of the emergence of a directed motion in nanosystems due to nonequilibrium fluctuations, is possible only in the presence of asymmetry, which can be caused by the asymmetry of either the stationary contribution to the potential energy or the fluctuation contribution, or by the relative shifts of the symmetry axes (centers) of these contributions. The latter possibility is preferable for optical lattices, periodic potential profiles created by strong electromagnetic fields in the optical range, since it is easier to create just symmetric lattices and then introduce asymmetry to the system by a phase shift between the stationary and fluctuating contributions. In this paper, we constructed a ratchet model controlled by such a phase shift. It differs from the known models of ratchet systems in that the stationary potential profile has the universal symmetry, and the fluctuating contribution is described by either symmetric or antisymmetric periodic functions. It turned out that, due to the universal symmetry of the stationary contribution, the dependence of the average velocity on the phase shift is also described by a periodic function of universal symmetry; the zeros of this function correspond to the ratchet stopping points, and the passage through these points ensures the motion reversal.

The average ratchet velocity depends on a set of parameters that characterise the stationary and fluctuating contributions to the potential energy, as well as on the fluctuation frequency and the ambient temperature. The time dependence of the fluctuations was taken into account through a symmetric dichotomous process, deterministic or stochastic. In the high-temperature motion mode, when the thermal energy exceeds the energy barriers of the potential relief, analytical expressions for the average velocity contain a factor that describes the time dependence of arbitrary fluctuations. Therefore, the analysis of the dependence of the obtained expression on the parameters of the potential profile is of the general nature.

The main result of the performed analysis is the additional stopping point found for the ratchet with the symmetric stepwise potential disturbed by the spatially symmetric fluctuations. A distinctive feature of this stopping point is the fact that the average velocity preserves its sign both to the left and to the right of it. Thus, the appearance of the positive curvature of the deformation of the stepwise potential shifts vertically the curve of the dependence of the average velocity on the phase shift, and this leads to the emergence of a pair of additional stopping points.

The optimal motion modes of the ratchet systems under consideration are those at which the greatest values of the average ratchet velocity can be achieved. One of such modes can be arranged at a large positive curvature of the deformation of the stepwise potential relief, when the symmetry axis of the fluctuating part of the potential energy coincides with the symmetry center of its stationary part. It is known that in various applications, not only large values of average velocities of particle motion are important, but also the possibility of controlling the motion direction. This applies to the particle separation in solutions and ion pumps [34; 35] and controlling vortices in superconducting systems [36], atoms in dissipative optical lattices [30–32], electrons in organic semiconductors [37], and molecular rotors [38; 39]. The factors that determine the drift direction of Brownian particles under the action of nonequilibrium fluctuations are the asymmetry of the spatio-temporal dependence of their potential energy, the phase shift between the stationary and fluctuating components of the potential energy, temperature, fluctuation frequency, and particle mass [9]. It turned out that, along with these factors, the curvatures of the flat sections of stationary contributions to the potential relief of universal symmetry under symmetric spatio-temporal fluctuations is another tool for controlling the motion direction.

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